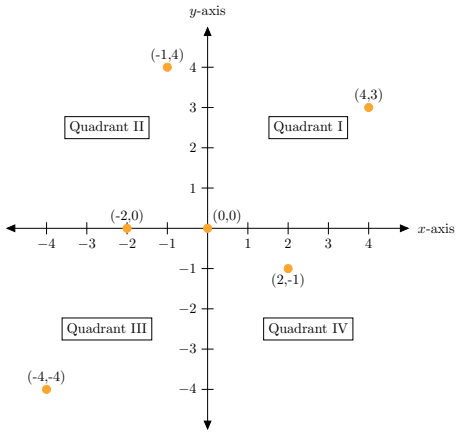
Section 1.1 Notes

Introduction to Graphing

1 Graphing

The Coordinate Plane

- The *Cartesian* or *coordinate plane* is a two-dimensional plane where the points are described using a pair of numbers (x, y). The first number gives the horizontal location and the second number gives the vertical location of the point.
- The *origin* is the name for the "center" at (0,0).



Solutions of Equations

- A *solution* to an equation with two variables (typically x and y) is an ordered pair (x, y). When the x-coordinate is plugged in for x and the y-coordinate is plugged in for y, the equation must be a true statement.
- For example: Consider the equation $3x - 1 = 2y$.
  - $(5, 7)$ is a solution because:

    $3(5) - 1 = 15 - 1 = 14$

    $2(7) = 14$

    $14 = 14$ ✓
  - $(1, -2)$ is *not* a solution because:

    $3(1) - 1 = 3 - 1 = 2$

    $2(-2) = -4$

    $2 \neq -4$ ✗
  - $(1, 1)$ is a solution because:

    $3(1) - 1 = 3 - 1 = 2$

    $2(1) = 2$

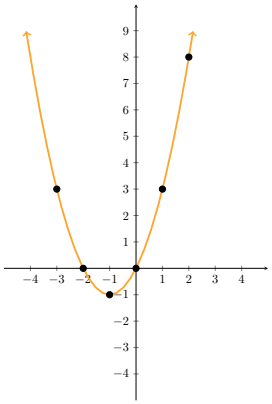
    $2 = 2$ ✓

Graphs of Equations

The *graph* of an equation is the visual representation of all the solutions to the equation. To find solutions, you plug in numbers for one variable and solve for the other variable.

Example: Graph $y = x^2 + 2x$.

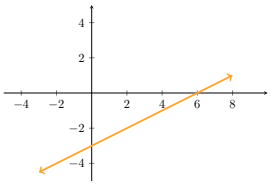
x	y
-3	$(-3)^2 + 2 \cdot (-3) = 3$
-2	$(-2)^2 + 2 \cdot (-2) = 0$
-1	$(-1)^2 + 2 \cdot (-1) = -1$
0	$0^2 + 2 \cdot 0 = 0$
1	$1^2 + 2 \cdot 1 = 3$
2	$2^2 + 2 \cdot 2 = 8$



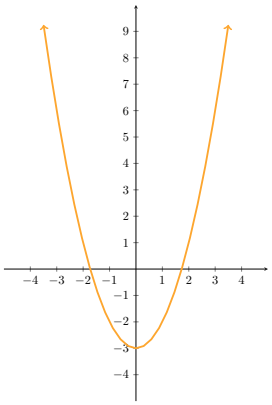
Examples

Graph the following equations.

1. $x - 2y = 6$

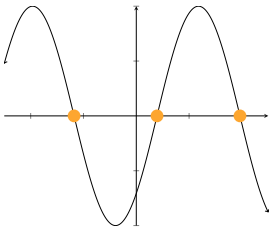


2. $y = x^2 - 3$



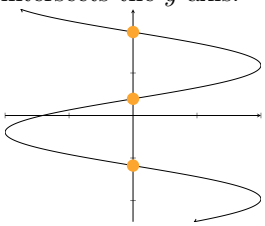
Intercepts

An *x-intercept* is a point where the graph intersects the x-axis. These are also called *roots* and *zeros*.



To find algebraically, set $y = 0$ and solve.

A *y-intercept* is a point where the graph intersects the y-axis.



To find algebraically, set $x = 0$ and solve.

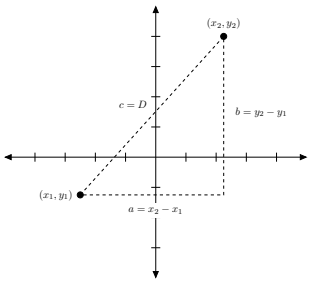
Example

Find the intercepts of $y = x^3 - x$.

x-intercepts: (0,0), (1,0), (-1,0)
y-intercept: (0,0)

2 Formulas

Distance Formula



The distance between (x_1, y_1) and (x_2, y_2) on the plane is given by the formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

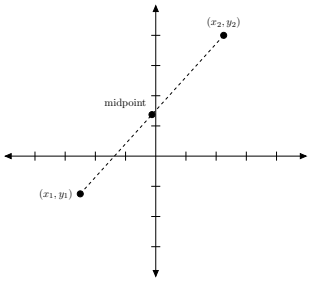
$$a^2 + b^2 = c^2 \text{ (Pythagorean Theorem)}$$
$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = D^2$$
$$\pm\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{D^2}$$
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = D \text{ (distance is always positive)}$$

Example

Find the distance between $(2, -4)$ and $(-1, -1)$.

$3\sqrt{2}$

Midpoint Formula



The midpoint between (x_1, y_1) and (x_2, y_2) on the plane is given by the formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Examples

1. Find the midpoint between $\left(\frac{1}{3}, -2\right)$ and $\left(\frac{4}{3}, -3\right)$.

   $\left(\frac{5}{6}, -\frac{5}{2}\right)$

2. If A has coordinates (1,2) and the midpoint between A and B is (-3,1), find B.

   $(-7, 0)$

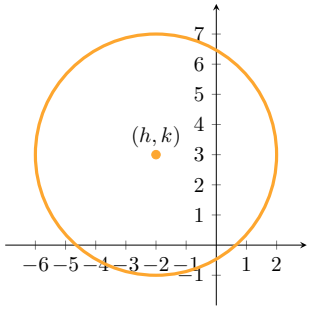
3 Circles

Formula

The equation of a circle is given by

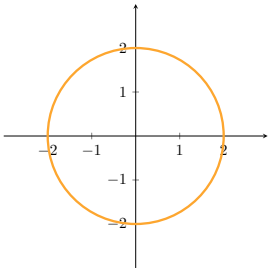
$$(x-h)^2+(y-k)^2=r^2$$

where (h,k) is the center of the circle and $r>0$ is the radius.

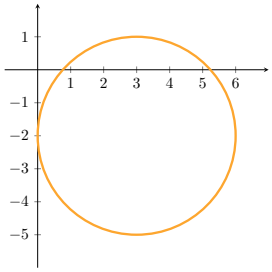


Examples

1. Graph $x^2+y^2=4$.



2. Graph $(x-3)^2+(y+2)^2=9$.



3. If a circle has a diameter with endpoints $(-2,4)$ and $(3,6)$, find the radius of the circle.

$$\frac{\sqrt{29}}{2}$$

4. If a circle has a diameter with endpoints $(-4,-5)$ and $(2,2)$, find the center of the circle.

$$\left(-1,-\frac{3}{2}\right)$$

5. Find the equation of a circle with center $(1,-3)$ and radius 5.

$$(x-1)^2+(y+3)^2=25$$