Section 1.2 Notes

Functions and Graphs

1 Introduction

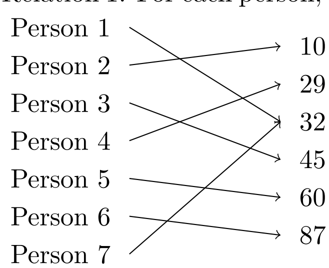
Definitions

- A *relation* is a rule that assigns inputs to outputs.
- A *function* is a relation where each input must have exactly one output - in other words, no single input can have two or more outputs.
- The *domain* of a function is the set of all inputs. Traditionally, these are represented by x-values.
- The *range* of a function is the set of all outputs. Traditionally, these are represented by y-values.

Functions versus General Relations

Some Made-Up People:
Person 1: Emma Johnson, 32 years old
Person 2: Liam Brown, 10 years old
Person 3: Olivia Miller, 45 years old
Person 4: William Davis, 29 years old
Person 5: Ava Garcia, 60 years old
Person 6: Sophia Wilson, 87 years old
Person 7: Willam Adams, 32 years old

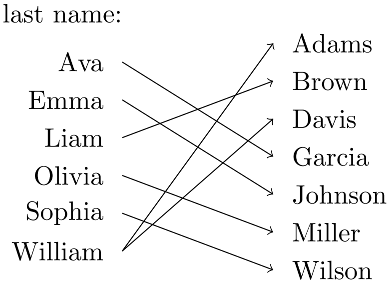
Relation 1: For each person, output their age:



Relation 1 *is* a function.

Some Made-Up People:
Person 1: Emma Johnson, 32 years old
Person 2: Liam Brown, 10 years old
Person 3: Olivia Miller, 45 years old
Person 4: William Davis, 29 years old
Person 5: Ava Garcia, 60 years old
Person 6: Sophia Wilson, 87 years old
Person 7: Willam Adams, 32 years old

Relation 2: For each first name, output their last name:



Relation 2 *is not* a function because of the first name “William”.

Examples

1. Is the relation a function? Find the domain and range.

$$\{(6,5),(8,2),(6,10)\}$$

No, it's not a function.
Domain: $\{6, 8\}$
Range: $\{5, 2, 10\}$

2. Is the relation a function? Find the domain and range.

$$\{(3, -1), (-11, 20), (0, 1)\}$$

Yes, it is a function.
Domain: $\{3, -11, 0\}$
Range: $\{-1, 20, 1\}$

Function Notation

- In this class, the functions we will be dealing with will be algebraic. The inputs will be numbers, the outputs will be numbers, and the rule describing how to get the outputs will be an algebraic formula.

- We use a special notation to write functions:

$$\underset{\text{function name}}{f}\,(\underset{\substack{\uparrow\\\text{input}\\\text{variable}}}{x}) = \underbrace{2x + 4}_{\text{formula}}$$

- "$f(x)$" is read as "f of x".

- $f(a)$ is the function f evaluated at the point $x = a$.

Examples

1. If $f(x) = \frac{2x-7}{x^2+1}$, find $f(3)$.

$-\frac{1}{10}$

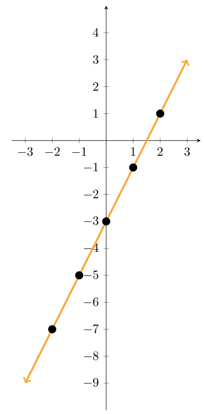
2. If $f(x) = x^3 - 2$, find $f(a + 2)$.

$a^3 + 6a^2 + 12a + 6$

2 Graphs of Functions

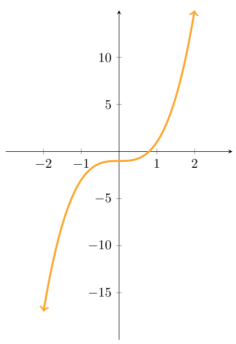
Graphing a Function
With functions, x is the input and $f(x)$ is the output. To graph, we can treat $f(x)$ just like y.
For example, graphing $f(x) = 2x - 3$ is exactly like graphing $y = 2x - 3$:

x	y
-2	-7
-1	-5
0	-3
1	-1
2	1

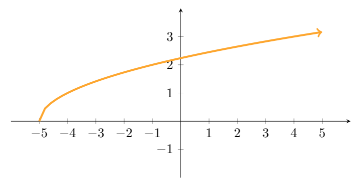


Examples

1. Graph $g(x) = 2x^3 - 1$.

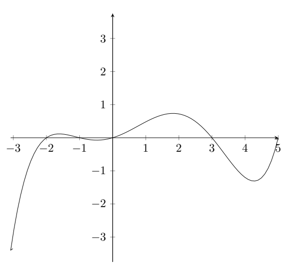


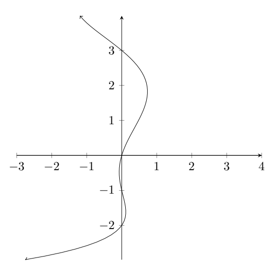
2. Graph $f(x) = \sqrt{x+5}$.

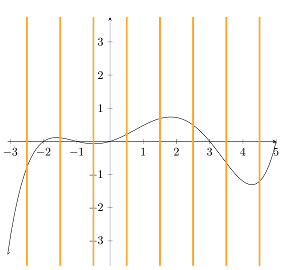


The Vertical Line Test

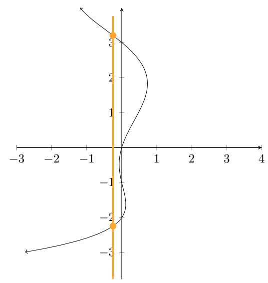
- Any equation with x and y can be graphed, but not all of them correspond to functions (with x as the input and y as the output).
- To be a function, each input can have just *one* output.
- *Vertical Line Test*: A graph passes the Vertical Line Test if every vertical line intersects the graph at most once.
- If the graph of an equation passes the Vertical Line Test, it *is* a function. If it fails, it's not.







This is a function.



This is not a function.

3 Domain and Range

Finding the Domain of an Algebraic Function
Finding the domain of a function works exactly like finding the domain of an expression:

1. If the expression contains $\boxed{\dfrac{A}{B}}$, then $\boxed{B \neq 0}$.
2. If the expression contains $\boxed{\sqrt{B}}$, then $\boxed{B \geq 0}$.

- If the expression contains $\boxed{\dfrac{A}{\sqrt{B}}}$, then $\boxed{B > 0}$. This rule *only* works when the *entire* denominator is under a square root.

Examples
Find the domains in interval notation:

1. $f(x) = x^3 - 3x + 1$

   $\boxed{(-\infty, \infty)}$

2. $g(x) = \frac{1}{x^2-16}$

   $\boxed{(-\infty, -4) \cup (-4, 4) \cup (4, \infty)}$

3. $h(x) = \sqrt{3x - 12}$

   $\boxed{[4, \infty)}$

4. $h(x) = \frac{2x+1}{\sqrt{5-x}}$

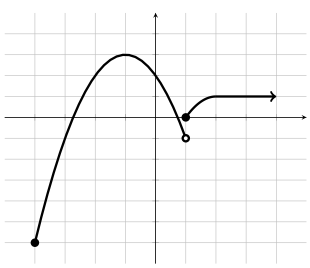
   $\boxed{(-\infty, 5)}$

Finding Domain and Range from a Graph

1. Finding the domain/x-values:
   - We need to write down (usually in interval notation) all of the x-values that show up on the graph.
   - Create a horizontal number line below the graph.
   - Fill in any points along this number line that have a corresponding point (same x-value) along the graph.
2. Finding the range/y-values:
   - We need to write down (usually in interval notation) all of the y-values that show up on the graph.
   - Create a vertical number line to the side of the graph.
   - Fill in any points along this number line that have a corresponding point (same y-value) along the graph.

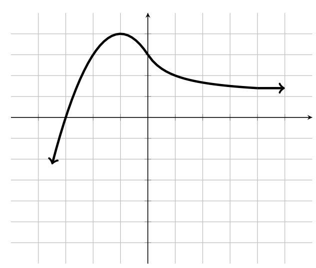
Examples
Find the domain and range of each.

1.



Domain: $[-4, \infty)$
Range: $[-6, 3]$

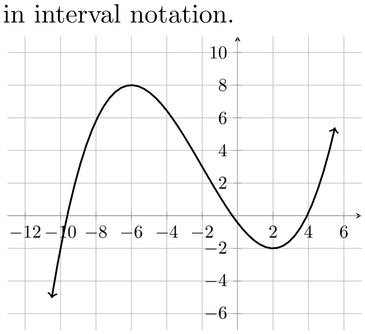
2.



Domain: $(-\infty, \infty)$
Range: $(-\infty, 4]$

Examples (contined)

3. The graph of $g(x)$ is given. Evaluate $g(-6)$, $g(-3)$, and $g(2)$. Then find the domain and range in interval notation.



$g(-6) = 8$
$g(-3) = 5$
$g(2) = -2$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$