Section 1.6 Notes

Solving Linear Inequalities

1 Introduction

Inquality Solving Techniques

1. You can add or subtract any number from both sides of an inequality.
2. You can multiply or divide both sides of an iequality by any positive number.
3. You can multiply or divide both sides of an inequality by any negative number, *however*, you must then flip the inequality.
   - "$<$" flips with "$>$"
   - "$\leq$" flips with "$\geq$"

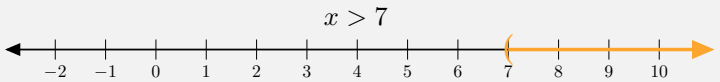
   Don't multiply both sides of an inequality by a variable! Since we don't know if the variable is positive or negative, we don't know whether to flip the inequality or not.

Examples

1. Solve the inequality and graph the solution.

$$4x - 7 > 2x + 7$$

$x > 7$



2. Solve the inequality and write the solution in interval notation.

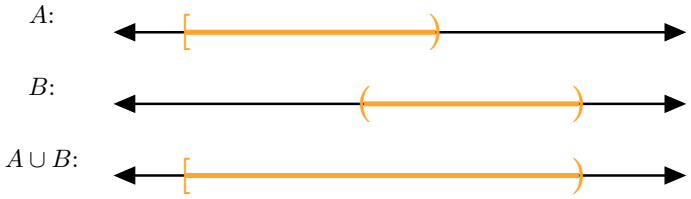
$$2(2x - 1)(x - 3) \leq 4x(x - 2)$$

$[1, \infty)$

2 Compound Inequalities

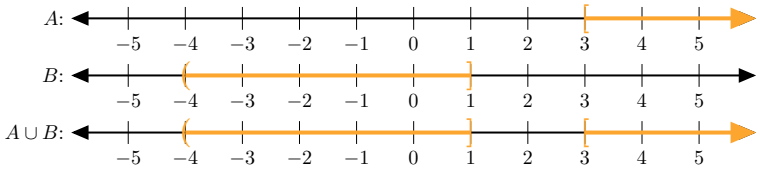
Union

If A and B are two sets, the *union* of A and B (denoted $A \cup B$) is the set containing any number that's in either A or B (or both).



The union notation is especially useful when we need to write an answer that includes multiple separate intervals.

For example:



We would write this as $(-4, 1] \cup [3, \infty)$.

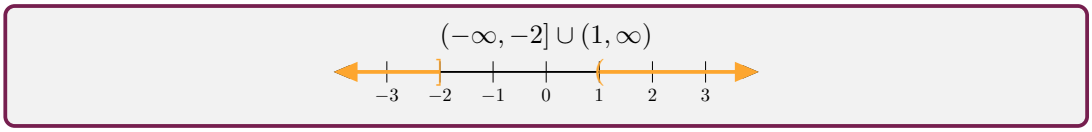
Disjuctions

- "*Disjunction*" is a fancy term for the word "or".
- For example, "$2x + 3 < 1$ or $5x - 6 \geq 2$" is a disjuction.
- To solve:
  - Solve each inequality separately - these will each give you an interval.
  - The final answer is the union of the two inequalities. Sometimes this can be simplified and written as a single interval.

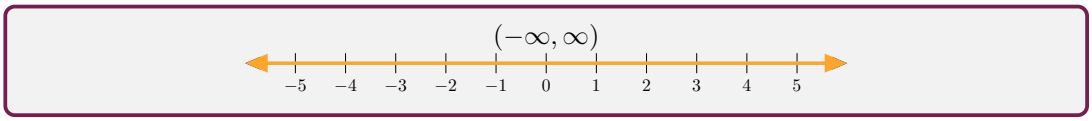
Examples

Solve the inequality, graph, and then write the answer in interval notation.

1. $3x \leq -6$ or $x - 1 > 0$



2. $2x + 3 < 5$ or $3x + 5 \geq 8$



Conjunctions

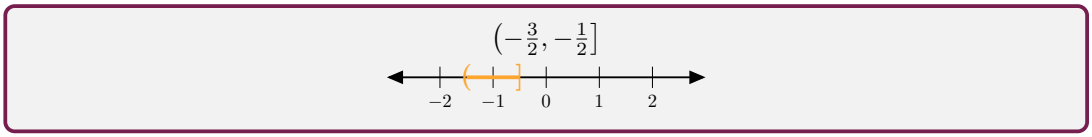
- "*Conjunction*" is a fancy term for the word "and".
- For example, "$2x + 3 < 1$ and $5x - 6 \geq 2$" is a conjuction.
- These come in two types:
  1. The "and" is explicitly written.
     - For example, "$4x + 6 > 8$ and $-x + 5 > x$"
     - To solve, solve each inequality separately.

- At the end, you're looking for the *overlap* of the two inequalities (graphing helps).

2. The "and" is implied.
   - For example, "$-1 < 5x + 6 < 7$" is a shorthand for "$-1 < 5x + 6$ and $5x + 6 < 7$".
   - To solve, whatever you do to one "part", you must do to the other two parts. Isolate the x in the middle part.

Examples
Solve the inequality, graph, and then write the answer in interval notation.

1. $6 \leq -2x + 5 < 8$



2. $5x \leq 10$ and $-2(x + 2) \leq -29$

No solution

3 Applications

Example

Karen can be paid in one of two ways for selling insurance policies:

- Plan A: A salary of \$750 per month, plus a commission of 10% sales;
- Plan B: A salary of \$1000 per month, plus a commission of 8% of sales in excess of \$2000

For what amount of monthly sales is plan A better than plan B if we can assume that sales are always more than \$2000?

She must sell more than \$4500.