Section 2.1 Notes

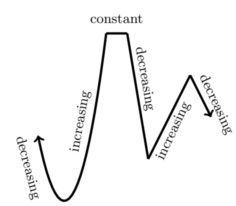
Increasing, Decreasing, and Piecewise Functions; Applications

1 Features of Graphs

Intervals of Increase and Decrease

- A function is *increasing* when the graph goes *up* as you travel along it from left to right.
- A function is *decreasing* when the graph goes *down* as you travel along it from left to right.
- A function is *constant* when the graph is a perfectly flat horizontal line.

  For example:

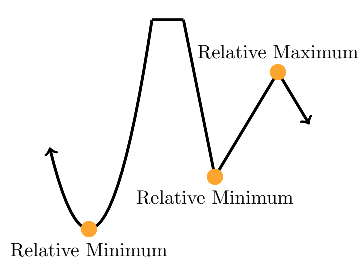


When we describe where the function is increasing, decreasing, and constant, we write *open* intervals written in terms of the x-values where the function is increasing or decreasing.

Relative Maximums and Minimums

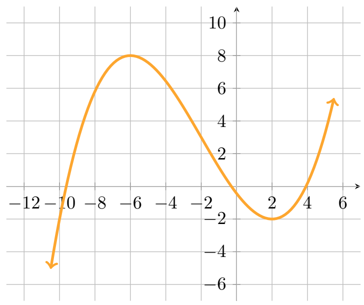
- *Relative maximums* (more properly maxima) are points that are higher than all nearby points on the graph.
- *Relative minimums* (more properly minima) are points that are lower than all nearby points on the graph.
- The phrase *relative extrema* refers to both relative maximums and minimums.

  For example:



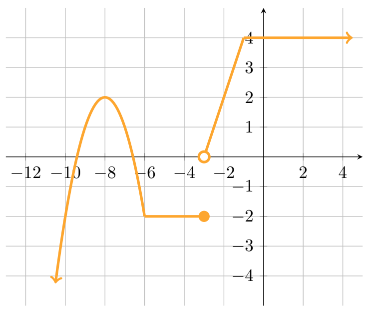
Examples

1. The graph of $g(x)$ is given. Find all relative maximums and minimums as well as the intervals of increase and decrease.



Relative Maximums:
8 at $x = -6$.
Relative Minimums:
-2 at $x = 2$.
Intervals of Increase:
$(-\infty, -6) \cup (2, \infty)$
Intervals of Decrease:
$(-6, 2)$

2. The graph of $h(x)$ is given. Find the intervals where $h(x)$ is increasing, decreasing and constant. Then find the domain and range.



Increasing: $(-\infty, -8) \cup (-3, -1)$
Decreasing: $(-8, -6)$
Constant: $(-6, -3) \cup (-1, \infty)$
Domain:$(-\infty, \infty)$
Range:$(-\infty, 4]$

2 Applications

Example

Creative Landscaping has 60 yd of fencing with which to enclose a rectangular flower garden. If the garden is x yards long, express the garden's area as a function of its length. Use a graphing device to determine the maximum area of the garden.

$A(x) = x(30 - x)$
Maximum Area: 225 square yards

3 Piecewise Functions

Definition
A *piecewise function* has several formulas to compute the output. The formula used depends on the input value.
For example,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Examples

If $h(t) = \begin{cases} 0 & \text{if } t < -2 \\ \frac{12t}{t-1} & \text{if } -2 \le t < 1 \\ 4t-3 & \text{if } t \ge 1 \end{cases}$, find

1. $h(0)$

   $\boxed{0}$

2. $h\left(\frac{4}{3}\right)$

   $\boxed{\frac{7}{3}}$

3. $h(-100)$

   $\boxed{0}$

Graphing

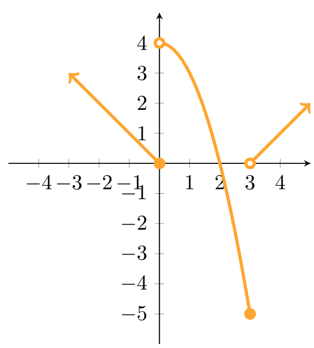
1. Split apart the function into the separate formulas. Determine what the graphs of each of those formulas looks like separately.
2. Use the inequalities to figure out what "section" you need from each graph.
3. Put all the "sections" together on a single graph, making sure to correctly plot the endpoints.
   - $<$ or $>$ - use an open circle
   - $\le$ or $\ge$ - use a closed circle

   None of the sections should have any vertical overlap! (Otherwise, it fails the vertical line test so what you've drawn isn't a function.)

Examples

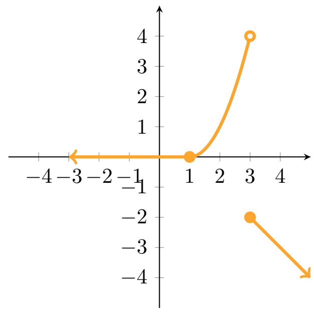
1. Graph

$$f(x) = \begin{cases} -x & \text{if} & x \le 0 \\ 4-x^2 & \text{if} & 0 < x \le 3 \\ x-3 & \text{if} & x > 3 \end{cases}$$



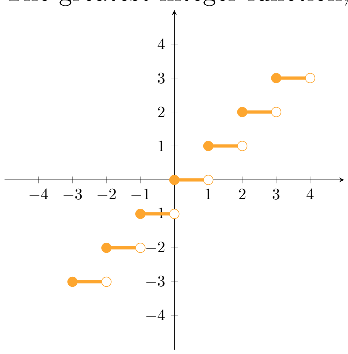
2. Graph

$$f(x) = \begin{cases} 0 & \text{if} & x \le 1 \\ (x-1)^2 & \text{if} & 1 < x < 3 \\ -x+1 & \text{if} & x \ge 3 \end{cases}$$



Greatest Integer Function

The greatest integer function, $y = [\![x]\!]$, rounds every number down to the nearest integer.



$$[\![x]\!] = \begin{cases} \vdots & \\ -2 & \text{if} \quad -2 \leq x < -1 \\ -1 & \text{if} \quad -1 \leq x < 0 \\ 0 & \text{if} \quad 0 \leq x < 1 \\ 1 & \text{if} \quad 1 \leq x < 2 \\ 2 & \text{if} \quad 2 \leq x < 3 \\ \vdots & \end{cases}$$

Examples

1. $[\![-22.5]\!]$

   $\boxed{-23}$

2. $[\![4.7]\!]$

   $\boxed{4}$

3. $[\![30]\!]$

   $\boxed{30}$

4. Find the range of values that x could be.

$$[\![x]\!]^2 = 16$$

$\boxed{4 \leq x < 5 \text{ or } -4 \leq x < -3}$