

Section 2.3 Notes

The Composition of Functions

1 Function Composition

Definition

- *Composition* is the process of plugging one function into another:

$$(f \circ g)(x) = f(g(x))$$

- For example: $f(x) = 2x - 1$ and $g(x) = x + 5$:

$$(f \circ g)(x) = f(x + 5) = 2(x + 5) - 1 = 2x + 10 - 1 = 2x + 9$$

- The domain can be found by looking at the *unsimplified* formula, and using our usual rules for finding domains.

Examples

1. Find $(f \circ g)(-2)$ if $f(x) = 3x + 1$ and $g(x) = \frac{2}{x}$.

-2

2. If $f(x) = \sqrt{x}$ and $g(x) = 2 - 3x$, find $f \circ g$, $g \circ f$, and their domains.

$$(f \circ g)(x) = \sqrt{2 - 3x}$$

$$\text{Domain: } (-\infty, \frac{2}{3}]$$

$$(g \circ f)(x) = 2 - 3\sqrt{x}$$

$$\text{Domain: } [0, \infty)$$

3. If $f(x) = \frac{6}{x}$ and $g(x) = \frac{x}{2x+1}$, find $g \circ f$, and its domain.

$$(g \circ f)(x) = \frac{6}{x + 12}$$

$$\text{Domain: } (-\infty, -12) \cup (-12, 0) \cup (0, \infty)$$

4. If $f(x) = 2x^2$ and $g(x) = \sqrt{2x - 4}$, find $f \circ g$ and its domain.

$$(f \circ g)(x) = 4x - 8$$

$$\text{Domain: } [2, \infty)$$

5. Find functions $f(x)$ and $g(x)$ so that $h(x) = (f \circ g)(x)$

$$h(x) = \frac{1}{(x-2)^4}$$

$$f(x) = \frac{1}{x}, g(x) = (x-2)^4$$

6. An oil spill has occurred offshore. The spill is circular in shape, and the radius is increasing at a rate of 2 meters per second.

- (a) Find $r(t)$, the formula for the radius of the spill at a time t (in seconds).

$$r(t) = 2t$$

- (b) Find $A(r)$, the formula for the area of the spill in terms of the radius.

$$A(r) = \pi r^2$$

- (c) Find $(A \circ r)(t)$.

$$(A \circ r)(t) = 4\pi t^2$$

- (d) Describe the meaning of $(A \circ r)(t)$.

The formula gives the area in terms of the time.