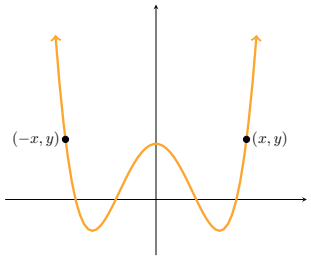
Section 2.4 Notes

Symmetry

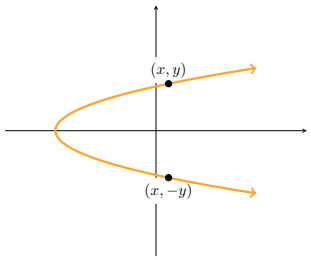
1 Types of Symmetry

Symmetry across the y-axis



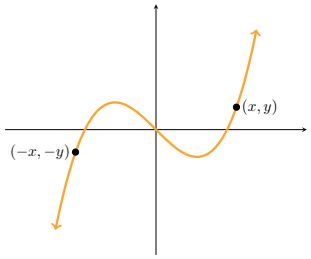
- The graph looks the same when flipped across the y-axis.
- To test for this type of symmetry:
  1. Replace all the x's with $-x$ and simplify.
  2. If the new equation will simplify back to the original equation, it's symmetric across the y-axis.

Symmetry across the x-axis



- The graph looks the same when flipped across the x-axis.
- To test for this type of symmetry:
  1. Replace all the y's with $-y$ and simplify.
  2. If the new equation will simplify back to the original equation, it's symmetric across the x-axis.

Symmetry About the Origin



- The graph looks the same when rotated 180° around the origin.
- To test for this type of symmetry:
  1. Replace all the x's with $-x$, and y's with $-y$, then simplify.
  2. If the new equation will simplify back to the original equation, it's symmetric about the origin.

Examples

1. Determine if $x^2 + y^2 = 1$ is symmetric across the x-axis, y-axis, or about the origin.

   across the x-axis, across the y-axis, about the origin

2. Determine if $y + x = x^3$ is symmetric across the x-axis, y-axis, or about the origin.

   about the origin

3. Find the point that is symmetric to $(-1, 3)$ across the x-axis, y-axis, and about the origin.

   across the x-axis: $(-1, -3)$
   across the y-axis: $(1, 3)$
   about the origin: $(1, -3)$

2 Symmetry of Functions

Even and Odd Functions
Functions that have symmetry are given special names.

- *Even functions* are symmetric across the y-axis.

  $$f(-x) = f(x)$$

- *Odd functions* are symmetric about the origin.

  $$f(-x) = -f(x)$$

- Symmetry across the x-axis isn't really interesting with functions. Almost any graph you can draw that's symmetric across the x-axis will fail the vertical line test and so it's not a function.

Examples
Check if the function is even, odd, or neither.

1. $f(x) = x^3 - \frac{1}{x}$

   odd

2. $f(x) = x^4 + 4x^2 - 3$

   even

3. $f(x) = x^3 - 2x^2$

   neither

4. $f(x) = \frac{x^2-3}{x^3+2x}$

   odd