

Section 2.5 Notes

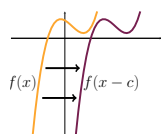
Transformations

1 Transformations

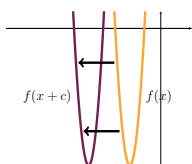
Translations

Horizontal Translations:

- The graph of $f(x - c)$ is $f(x)$ shifted c units to the right.

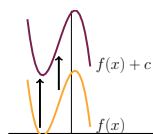


- The graph of $f(x + c)$ is $f(x)$ shifted c units to the left.

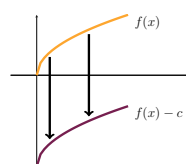


Vertical Translations:

- The graph of $f(x) + c$ is $f(x)$ shifted c units up.

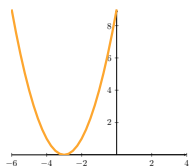
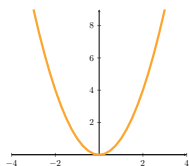


- The graph of $f(x) - c$ is $f(x)$ shifted c units down.

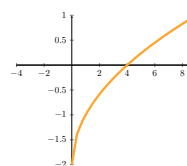
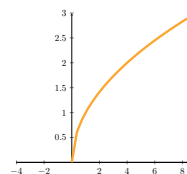


Examples

1. Graph $y = x^2$ and $y = (x + 3)^2$.



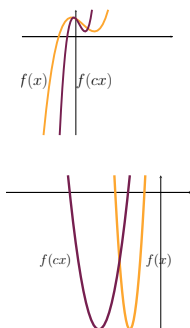
2. Graph $y = \sqrt{x}$ and $y = \sqrt{x} - 2$



Stretching/Shrinking

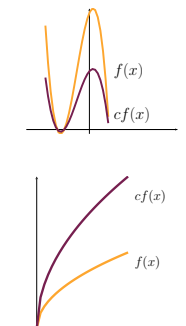
Horizontal Stretch/Shrink:

- If c is positive, $f(cx)$ is $f(x)$ stretched or shrunk by a factor of $\frac{1}{c}$ from/towards the y -axis.



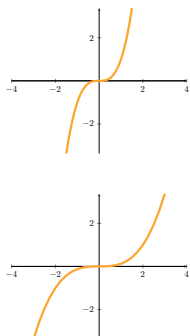
Vertical Stretch/Shrink:

- If c is positive, $cf(x)$ is $f(x)$ stretched or shrunk by a factor of c from/towards the x -axis.

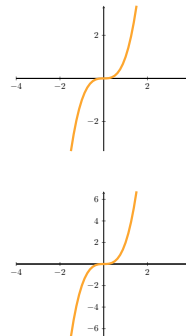


Examples

1. Graph $y = x^3$ and $y = (\frac{1}{2}x)^3$.



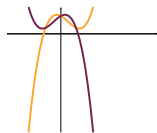
2. Graph $y = x^3$ and $y = 2x^3$.



Reflection

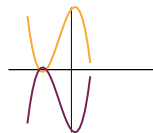
Horizontal Reflection:

- $f(-x)$ is $f(x)$ reflected across the y -axis.



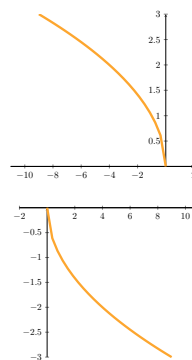
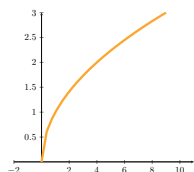
Vertical Reflection:

- $-f(x)$ is $f(x)$ reflected across the x -axis.

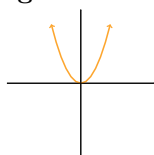


Examples

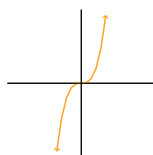
Graph $y = \sqrt{x}$ and $y = \sqrt{-x}$ and $y = -\sqrt{x}$.



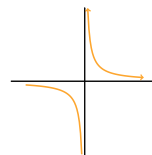
“Starting” Functions



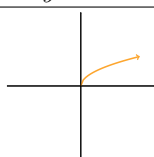
$$y = x^2$$



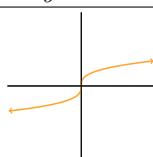
$$y = x^3$$



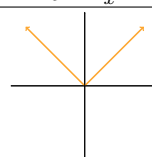
$$y = \frac{1}{x}$$



$$y = \sqrt{x}$$



$$y = \sqrt[3]{x}$$



$$y = |x|$$

Graphing Equations with Multiple Transformations

Let's say we want to graph $y = \frac{1}{2}\sqrt{4-x} + 1$

1. Identify the “starting” function.

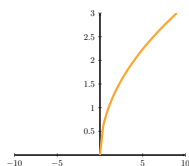
$$y = \sqrt{x}$$

2. Compare the “starting” function with the final function. Anything that's changed where the x is in the starting function will correspond to a horizontal transformation. Anything else will correspond to a vertical transformation.

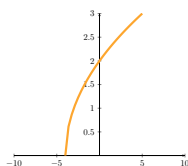
$$y = \sqrt{x} \quad \text{vs} \quad y = \frac{1}{2}\sqrt{4-x} + 1$$

Vertical
↑
Horizontal

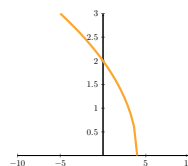
3. Graph the horizontal transformations, by going *against* the order of operations.



$$y = \sqrt{x}$$



$$y = \sqrt{x+4} = \sqrt{4+x}$$

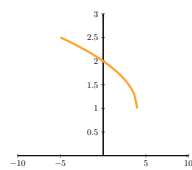


$$y = \sqrt{4-x}$$

4. Graph the vertical transformations, by following the order of operations.



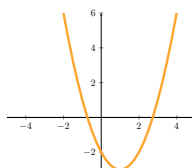
$$y = \frac{1}{2}\sqrt{4-x}$$



$$y = \frac{1}{2}\sqrt{4-x} + 1$$

Examples

1. Graph $y = (x - 1)^2 - 3$



2. Graph $y = -2\sqrt{x + 2}$

