

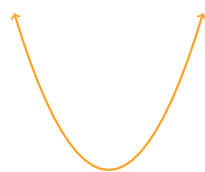
Section 3.3 Notes

Analyzing Graphs of Quadratic Functions

1 Introduction

Definitions

- A *quadratic function* is a function with the form $f(x) = ax^2 + bx + c$, where $a \neq 0$.
- The graphs of quadratic functions are all *parabolas* - informally, they have a “bowl” or a “U” shape, either upside down or right-side up.



$$a > 0$$

We say the parabola *opens up*.



$$a < 0$$

We say the parabola *opens down*.

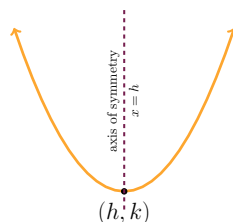
2 Vertex Form of a Quadratic

Definitions

- Every quadratic can be written in *vertex form*:

$$f(x) = a(x - h)^2 + k$$

- In this form, (h, k) is the *vertex* of the parabola (and a still determines if the parabola opens up or down):



- The parabola is always symmetric across the line $x = h$, which is the vertical line that goes through the vertex.

Putting an Equation into Vertex Form

The vertex of the parabola $f(x) = ax^2 + bx + c$ is given by:

$$h = -\frac{b}{2a}$$
$$k = f(h)$$

1. Calculate h and k using the formulas, and get a from the original equation.
2. Plug these into $f(x) = a(x - h)^2 + k$

Examples

Write the following quadratic functions into vertex form:

1. $f(x) = x^2 - 4x + 5$

$$f(x) = (x - 2)^2 + 1$$

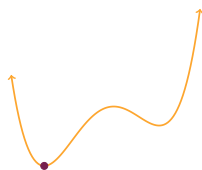
2. $f(x) = -3x^2 + 4x + 1$

$$f(x) = -3\left(x - \frac{2}{3}\right)^2 + \frac{7}{3}$$

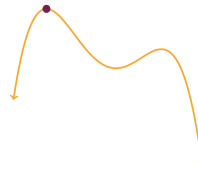
3 Maximums and Minimums of Quadratics

Absolute Maximums and Minimums

- An *absolute maximum* or *maximum* is a point on the graph that is higher than every other point.
- An *absolute minimum* or *minimum* is a point on the graph that is lower than every other point.



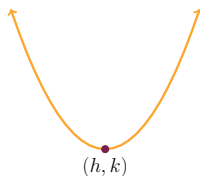
This has an absolute minimum but no absolute maximum.



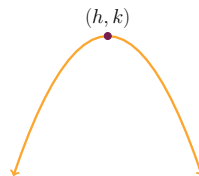
This has an absolute maximum but no absolute minimum.

Maximums and Minimums on a Quadratic

Every quadratic has either a maximum or a minimum at its vertex.



If $a > 0$, it has a minimum.



If $a < 0$, it has a maximum.

Examples

1. Mendoza Manufacturing plans to produce a one-compartment vertical file by bending the long side of a 10-in by 18-in. sheet of plastic along two lines to form a $\sqcup$ shape. How tall should the file be in order to maximize the volume it can hold, and what is the maximum volume?

A height of 4.5 in gives the files its maximum volume of 405 in^3 .

2. A soft-drink vendor determines its revenue and costs are determined by the function

$$R(x) = 10x$$

$$C(x) = 0.002x^2 + 2.4x + 120$$

where x is the number of drinks sold. Find the vendor's maximum profit.

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4 Graphing Quadratics

Method

1. Find the vertex of the parabola.
2. Find the x and y intercepts.
3. Plot all of the above points and connect them with a parabola.

Example

For the function $f(x) = 2x^2 - 7x - 4$, find the vertex, the x and y -intercepts, the axis of symmetry, the maximum or minimum, and then graph.

Vertex: $(\frac{7}{4}, -\frac{81}{8})$

x -intercepts: $(-\frac{1}{2}, 0), (4, 0)$

y -intercept: $(0, -4)$

Axis of symmetry: $x = \frac{7}{4}$

Minimum: $-\frac{81}{8}$

