

Section 3.5 Notes

Solving Equations and Inequalities with Absolute Value

1 Absolute Value Equations

Method

To solve, put the equation into the standard form by isolating the absolute value on one side of the equation. It should look like

$$|X| = c$$

- If $c < 0$, there are no solutions.
- If $c \geq 0$, solve the equations by setting $X = c$ and $X = -c$.

Examples

Solve the equations:

1. $|2x - 1| - 5 = -3$

$$x = \frac{3}{2} \text{ or } x = -\frac{1}{2}$$

2. $9 - |x - 2| = 11$

No solution.

2 Absolute Value Inequalities

Method

To solve, put the inequality into the standard form by isolating the absolute value on one side of the inequality.

Standard Form	$c > 0$ (normal case)	$c < 0$	$c = 0$
$ X < c$	$-c < X < c$	no solution	no solution
$ X \leq c$	$-c \leq X \leq c$	no solution	$X = 0$
$ X > c$	$X < -c$ or $X > c$	all real numbers	all real numbers except $X = 0$
$ X \geq c$	$X \leq -c$ or $X \geq c$	all real numbers	all real numbers

Examples

Solve the inequalities and write the answers in interval notation:

1. $4 - |2x + 3| \leq 3$

$$(-\infty, -2] \cup [-1, \infty)$$

2. $1 - |3x + 4| > 9$

No solution.

3. $|6 - 4x| \leq 8$

$$\left[-\frac{1}{2}, \frac{7}{2}\right]$$

4. $3|x + 4| + 2 > 1$

$$(-\infty, \infty)$$