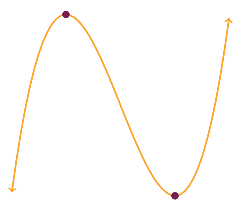
Section 4.2 Notes

Graphing Polynomial Functions

1 Features of Polynomial Graphs

Definitions

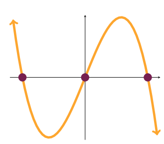
- Previously, we've said that x-intercepts and zeros are essentially the same thing. Now that we've introduced complex numbers, there is a slight difference:
  - A *zero* is *any* x-value you solve for when $y = 0$ (real or complex).
  - An *x-intercept* only includes real-valued solutions when you plug in $y = 0$.
- A *turning point* on the graph is a point where the graph switches from increasing to decreasing or from decreasing to increasing. These are just another name for the relative maximum and minimum points.



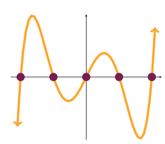
Features Based on the Degree

- If a polynomial has degree n, then it has *at most* n x-intercepts/real-valued zeros (there might be fewer than n)

  For example a polynomial with degree 3
  - might look like this:

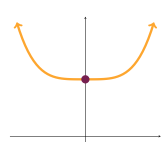


  - but can't look like this:

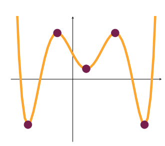


- If a polynomial has degree n, then it has *at most* $n - 1$ turning points.

  For example a polynomial with degree 4
  - might look like this:



  - but can't look like this:



2 Graphing Polynomial Functions

Goal
When we draw our graph, we want to have two features correct:

1. The end behavior

2. The behavior near the roots

   Other details might be slightly incorrect, but we only need those two aspects right. For a better graph, you'd need to plug in a large number of points.

Method

1. Determine both the leading term and the factored form of the polynomial.

2. Use the factored form to find and plot the zeros.

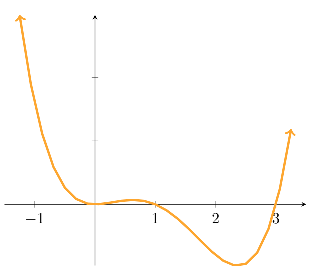
3. Plug in and plot any extra points you want - y-intercept is nice. This step is optional.

4. Use the leading term to determine the end behavior. On the graph, draw the two sides onto the outer two most zeros.

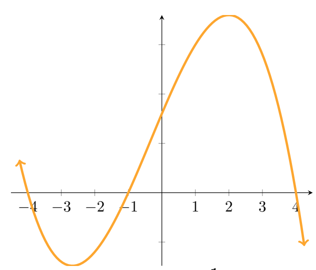
5. Use the factored form to find the multiplicities of the zeros. Use these to fill in the "middle" part of the graph.

Examples

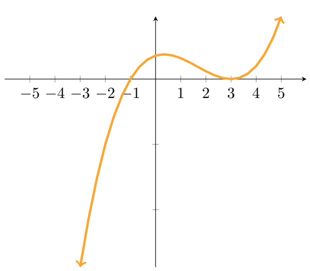
1. Graph $P(x) = x^4 - 4x^3 + 3x^2$



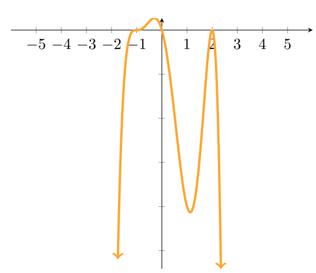
2. Graph $P(x) = -2x^3 - 2x^2 + 32x + 32$



3. Graph $P(x) = (x+1)(2x^2 - 12x + 18)$



4. Graph $P(x) = -\frac{1}{4}x(x+1)^3(x-2)^2$



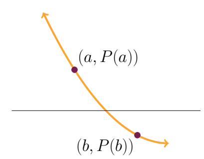
3 Intermediate Value Theorem

Explanation

- Remember that polynomial graphs are continuous - they can be drawn in a single piece with no jumps/breaks/holes.

- The Intermediate Value Theorem helps us find real-valued zeros/x-intercepts:

  > Suppose $P(x)$ is a polynomial with real number coefficients, and a and b are two different x-values. Find the y-values $P(a)$ and $P(b)$. When these are on opposite sides of the x-axis, then somewhere between a and b is an x-intercept.



Example

Use the Intermediate Value Theorem to determine if $g(x) = 3x^3 - 14x^2 - 9x + 20$ has a real zero between -2 and -1.

> Yes, it has a real zero betwen -2 and -1.