Section 4.6 Notes

Polynomial Inequalities and Rational Inequalities

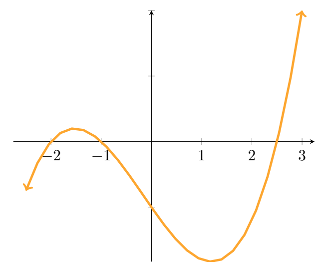
1 Polynomial Inequalities

Method 1: Graphing
Let's work through the process using $(x+1)(x+2)(2x-5) < 0$.

1. Add/subtract terms to get zero on the right-hand-side (RHS) of the inequality.
2. Factor the polynomial on the left-hand-side (LHS) and then graph it.

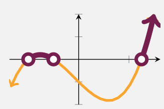
   Leading Coefficient: $2x^3$
   Zeros:

   - -2 with multiplicity 1
   - -1 with multiplicity 1
   - $\frac{5}{2} = 2.5$ with multiplicity 1

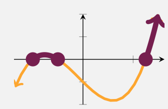


3. Read the answer off of the graph:

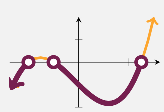
$f(x) > 0$ is asking where the graph is above the x-axis.



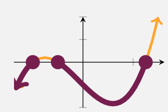
$f(x) \geq 0$ is asking where the graph is at or above the x-axis.



$f(x) < 0$ is asking where the graph is below the x-axis.

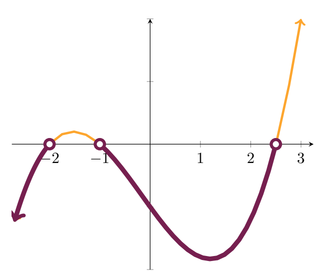


$f(x) \leq 0$ is asking where the graph is at or below the x-axis.



For our example, we're looking at

$$(x+1)(x+2)(2x-5) < 0$$



Answer: $\boxed{(-\infty, -2) \cup \left(-1, \frac{5}{2}\right)}$

Method 2: Numerical

Let's work through the process using $x^2 - x - 13 \geq 2x + 5$.

1. Add/subtract terms to get zero on the right-hand-side (RHS) of the inequality.

$$x^2 - 3x - 18 \geq 0$$

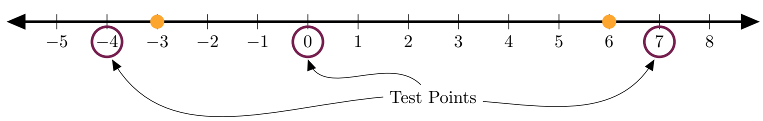
2. Factor the polynomial on the left-hand-side (LHS).

$$(x-6)(x+3) \geq 0$$

3. Set the factors on the LHS equal to zero and solve.

$$\begin{array}{ccc} x-6=0 & \text{or} & x+3=0 \\ x=6 & \text{or} & x=-3 \end{array}$$

4. Plot the values from step 3 on a number line and pick *test points* on each side and between these numbers.



- If the inequality is "$<$" or "$>$", plot them with open circles
- If the inequality is "$\leq$" or "$\geq$", plot them with closed circles

5. Check to see if each interval is "good" or "bad" using the test points.

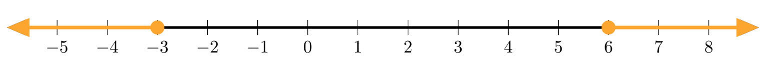
$$(x-6)(x+3) \geq 0$$

$x = -4:\quad (-4-6)(-4+3) = (-10)(-1) = 10 \geq 0$ ✓

$x = 0:\quad (0-6)(0+3) = (-6)(3) = -18 \geq 0$ ✗

$x = 7:\quad (7-6)(7+3) = (1)(10) = 10 \geq 0$ ✓

6. Fill in the "good" intervals and read the answer off the number line.



Answer: $\boxed{(-\infty, -3] \cup [6, \infty)}$

Examples

Solve the polynomial inequality and write the answer in interval notation:

1. $4x^3 - 4x^2 - x + 1 < 0$

$\boxed{\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right)}$

2. $x^2 - x - 5 \geq x - 2$

$\boxed{(-\infty, -1] \cup [3, \infty)}$

2 Rational Inequalities

Method

Let's work through the process using $\dfrac{3x+2}{2+x} < 2$.

1. Add/subtract terms to get zero on the right-hand-side (RHS) of the inequality and simplify to one fraction.

$$\frac{3x+2}{2+x} - 2 < 0$$
$$\frac{3x+2}{2+x} - \frac{2\cdot(2+x)}{1\cdot(2+x)} < 0$$
$$\frac{3x+2-4-2x}{2+x} < 0$$
$$\frac{x-2}{x+2} < 0$$

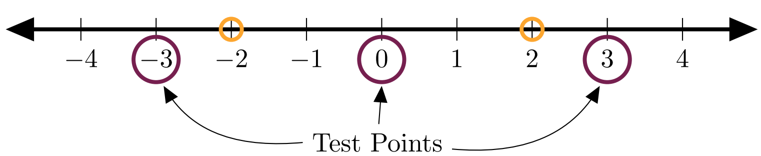
2. Factor the top and bottom of the fraction.

$$\frac{x-2}{x+2} < 0$$

3. Set all factors on top and bottom equal to zero and solve.

Top:	Bottom:
$x-2=0$ $x=2$	$x+2=0$ $x=-2$

4. Plot the values from step 3 on a number line and pick *test points* on each side and between these numbers.



- For values from the bottom, *always* use open circles.
- On top values, plot them with open circles for "<" or ">".
- On top values, plot them with closed circles for "≤" or "≥".

5. Check to see if each interval is "good" or "bad" using the test points.

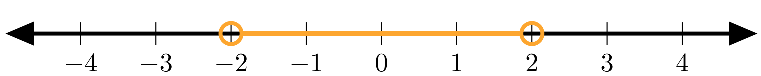
$$\frac{x-2}{x+2} < 0$$

$x=-3: \quad \frac{-3-2}{-3+2} = \frac{-5}{-1} = 5 < 0$ ✗

$x=0: \quad \frac{0-2}{0+2} = \frac{-2}{2} = -1 < 0$ ✓

$x=3: \quad \frac{3-2}{3+2} = \frac{1}{5} < 0$ ✗

6. Fill in the "good" intervals and read the answer off the number line.



Answer: $\boxed{(-2,2)}$

Example

Solve the polynomial inequality and write the answer in interval notation:

1. $\dfrac{x-3}{x+4} \leq \dfrac{x+2}{x-5}$

   $$\boxed{\left(-4, \tfrac{1}{2}\right] \cup (5, \infty)}$$

2. $\dfrac{(x-3)^2(x+1)}{(x-5)} \geq 0$

   $$\boxed{(-\infty, -1] \cup \{3\} \cup (5, \infty)}$$