

Section 5.2 Notes

Exponential Functions and Graphs

1 Graphing

Definition

- The *exponential function* with base a is given by

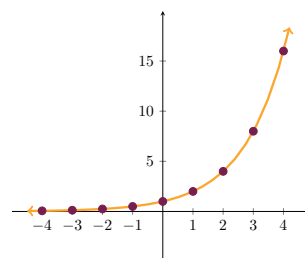
$$f(x) = a^x$$

- Here a must be positive, and also $a \neq 1$

Graphing Exponential Functions

- Let's graph $y = 2^x$

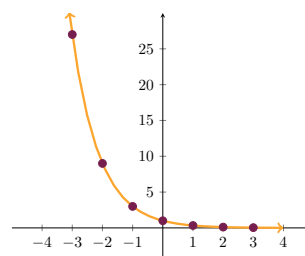
x	y
-4	$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$



Notice that the graph has a horizontal asymptote along the x -axis.

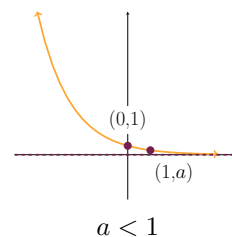
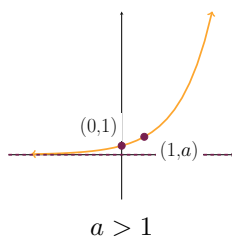
- Let's graph $y = \left(\frac{1}{3}\right)^x$

x	y
-3	$\left(\frac{1}{3}\right)^{-3} = 3^3 = 27$
-2	$\left(\frac{1}{3}\right)^{-2} = 3^2 = 9$
-1	$\left(\frac{1}{3}\right)^{-1} = 3^1 = 3$
0	$\left(\frac{1}{3}\right)^0 = 1$
1	$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$
2	$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$
3	$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$



Again, notice that the graph has a horizontal asymptote along the x -axis.

- In general, there are just two basic shapes for $f(x) = a^x$:



- Again, they both have horizontal asymptote along the x -axis.
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$

The Natural Exponential Function

- The *natural exponential function* is given by

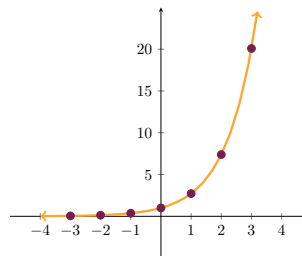
$$f(x) = e^x$$

- e is an irrational number

$$e \approx 2.7182818$$

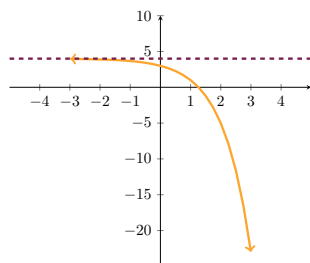
- Let's graph $y = e^x$

x	y
-3	$e^{-3} \approx 0.0498$
-2	$e^{-2} \approx 0.1353$
-1	$e^{-1} \approx 0.3679$
0	$e^0 = 1$
1	$e^1 \approx 2.7183$
2	$e^2 \approx 7.3891$
3	$e^3 \approx 20.0855$

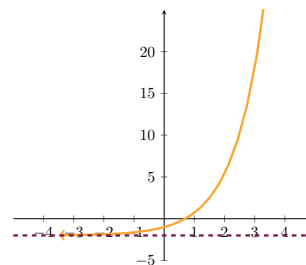


Examples

1. $y = 4 - 3^x$



2. $y = e^x - 2$



2 Modeling

Example

A company begins an Internet advertising campaign to market a new telephone. The percentage of the target market that buys a product is generally a function of the length of the advertising campaign. The estimated percentage is given by the exponential function

$$f(t) = 100(1 - e^{-0.04t}),$$

where t is the number of days of the campaign. Find $f(25)$, the percentage of the target market that has bought the product after a 25-day advertising campaign.

63.2%