

Section 5.4 Notes

Properties of Logarithmic Functions

1 Properties of Logarithms

Formulas

Basic Properties:

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a a^x = x$
- $a^{\log_a x} = x$

- The Product Rule:

$$\log_a(xy) = \log_a x + \log_a y$$

- The Quotient Rule:

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

- The Power Rule:

$$\log_a(x^z) = z \log_a x$$

Examples

Calculate the following:

1. $\log_4 64$

3

2. $10^{\log 51}$

51

3. $\log_3 \sqrt{27}$

$\frac{3}{2}$

4. $\ln e^{200}$

200

Use the Properties of Logarithms to expand the expression into sums and differences of logarithms:

5. $\log_2 \frac{2x^4}{y^2 z^{10}}$

$$1 + 4 \log_2 x - 2 \log_2 y - 10 \log_2 z$$

6. $\log_4 \sqrt[3]{x^2 - 2x}$

$$\frac{1}{3} \log_4 x + \frac{1}{3} \log_4 (x - 2)$$

7. $\log_5 \sqrt{\frac{3x}{y}}$

$$\frac{1}{2} \log_5 3 + \frac{1}{2} \log_5 x - \frac{1}{2} \log_5 y$$

8. Given that $\log_a 2 \approx 0.301$ and $\log_a 7 \approx 0.845$, find $\log_a 28$.

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Use the Properties of Logarithms to combine the expression into a single logarithm:

9. $-2 \ln 3 + 4 \ln x - \frac{1}{2} \ln(x-1)$

$$\ln \frac{x^4}{9\sqrt{x-1}}$$

10. $\log 4900 - 2 \log 7$

$$2$$

11. $4 \log_5(x-4) + \log_5(x-1) - 3 \log_5(x^2 - 5x + 4)$

$$\log_5 \frac{x-4}{(x-1)^2}$$