Section 6.1 Notes

Systems of Equations in Two Variables

1 Introduction

Definitions

- A *system of equations* is a list of two or more equations.
- A *linear system of equations* has only linear equations in the list.

  For example:

$$\begin{aligned} 2x - y &= 4 \\ x + y &= 2 \end{aligned}$$

- A *solution* to a system of equations is a pair of x and y-values that, when plugged in, make all the equations true.
- If a system has more than two variables, every solution consists of a number assigned to each variable.

  For example, the solution to the above system is $(2, 0)$ or $x = 2$, $y = 2$ because:

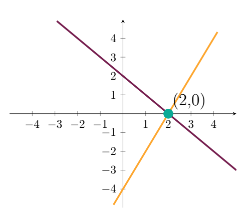
$$\begin{aligned} 2(2) - 0 &= 4 \checkmark \\ 2 + 0 &= 2 \checkmark \end{aligned}$$

2 Solving Systems by Graphing

- We can graph the equations in a system on a single coordinate plane.
- When we do this, the point(s) where the lines/curves cross is/are the solution(s).

  For example:

$$\begin{aligned} 2x - y = 4 &\to -y = -2x + 4 \to \boxed{y = 2x - 4} \\ x + y = 2 &\to \boxed{y = -x + 2} \end{aligned}$$



Example

Solve the system of equation by graphing:

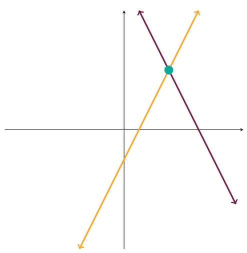
$$\begin{aligned} y &= x + 2 \\ y &= 2x + 5 \end{aligned}$$

$\boxed{(-3, -1)}$

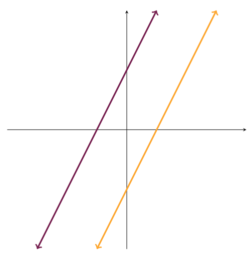
3 The Number of Solutions of a Linear System

Possible Graphs

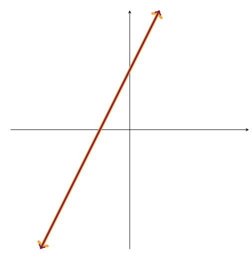
There are three possible cases for the number of solutions a linear system of two variables and two equations has:



The two lines cross at a single point.
One solution.



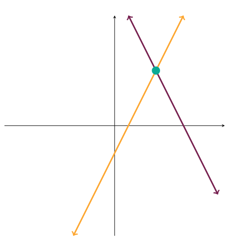
The two lines are parallel and never cross.
No solutions.



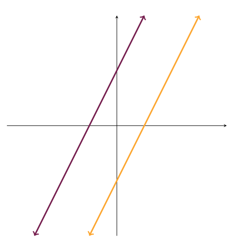
The two equations actually represent the same line.
Infinitely many solutions.

Definitions

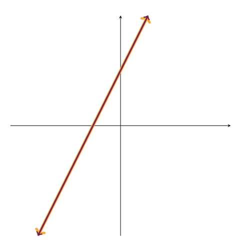
- A system is *consistent* if it has at least one solution.
- A system is *inconsistent* if it has no solutions.
- A linear system with two equations is *dependent* when one of the equations simplifies to the other.
- A linear system is *independent* when it's not dependent.



Consistent
Independent



Inconsistent
Independent



Consistent
Dependent

4 Algebraic Methods of Solving Systems

Substitution
Let's solve this system:

$$x - y = 1$$
$$4x + 3y = 18$$

1. Pick one of the two equations, and solve for either of the variables in this equation.

   Equation 1: $x = y + 1$

2. Plug this back into the other equation and solve.

$$\begin{aligned} 4x + 3y &= 18 \\ 4(y+1) + 3y &= 18 \\ 4y + 4 + 3y &= 18 \\ 7y &= 14 \\ y &= 2 \end{aligned}$$

3. Plug back in to any of the equations and solve for the final variable.

$$\begin{aligned} x &= y + 1 \\ x &= 2 + 1 \\ x &= 3 \end{aligned}$$

   The solution is $\boxed{(3,2).}$

4. In step 2, if you didn't get $x =\#$ or $y =\#$:
   - The equation simplified to something true (3=3). In this case you have infinitely many solutions. You need to write a formula for every possible solution as an ordered pair.
   - The equation simplified to something false (1=7). In this case you have no solutions.

Elimination
Let's solve this system:

$$3x - 5y = -11$$
$$4x + 2y = -6$$

1. Pick a variable to eliminate.

   We'll get rid of y.

2. Multiply the equations by appropriate numbers to get the coefficient of your chosen variable to match *but with opposite signs.*

$$2(3x - 5y) = 2(-11) \to 6x - \boxed{10}\,y = -22$$
$$5(4x + 2y) = 5(-6) \to 20x + \boxed{10}\,y = -30$$

3. Add two equations and solve.

$$\begin{array}{rrcr} 6x & -\cancel{10y} & = & -22 \\ 20x & +\cancel{10y} & = & -30 \\ \hline 26x & & = & -52 \end{array}$$

$x = -2$

4. Plug back in to any of the equations and solve for the final variable.

$$\begin{aligned} 3x - 5y &= -11 \\ 3(-2) - 5y &= -11 \\ -6 - 5y &= -11 \\ -5y &= -5 \\ y &= 1 \end{aligned}$$

The solution is $\boxed{\text{(-2,1)}}$.

5. Again, in step 3, if you didn't get $x =\#$ or $y =\#$:

   - The equation simplified to something true (3=3). In this case you have infinitely many solutions. You need to write a formula for every possible solution as an ordered pair.
   - The equation simplified to something false (1=7). In this case you have no solutions.

Examples

1.

$$\begin{aligned} 5x + 10y &= 7 \\ x + 2y &= -3 \end{aligned}$$

$\boxed{\text{No solutions}}$

2.

$$\begin{aligned} -4x + 3y &= 0 \\ 3x + 4y &= \frac{25}{4} \end{aligned}$$

$\boxed{\left(\frac{3}{4}, 1\right)}$

3.

$$\begin{aligned} 2x - y &= 6 \\ 6x &= 3y + 18 \end{aligned}$$

$\boxed{(x, 2x - 6) \text{ or } \left(\frac{1}{2}y + 3, y\right)}$

4. One serving of tomato soup contains 100 calories and 18 grams of carbohydrates. One slice of whole wheat bread contains 70 calories and 13 grams of carbohydrates. How many servings of each would be required to obtain 230 calories and 42 grams of carbohydrates?

   > 1.25 servings of tomato soup
   > 1.5 servings of whole wheat bread

5. One evening 1500 concert tickets were sold for the Fairmont Summer Jazz Festival. Tickets cost \$25 for a covered pavilion seat and \$15 for a lawn seat. Total receipts were \$28,500. How many of each type of ticket were sold?

> 600 pavilion seats
> 900 lawn seats