Section 6.2 Notes

Systems of Equations in Three Variables

Number of Solutions of a Linear System
Like the two variable case, when you have systems of more variables:

- You can have exactly one solution
- You can have no (zero) solutions
- You can have infinitely many solutions.

Substitution
Let's solve this system:

$$\begin{aligned} x - y + z &= 0 \\ y + 2z &= -2 \\ x + y - z &= 2 \end{aligned}$$

1. Pick one equation and solve for a variable in that equation.

   Equation 2: $y + 2z = -2 \rightarrow y = -2z - 2$

2. Plug that into *both* of the other equations.

$$\begin{aligned} x - (-2z - 2) + z &= 0 \\ x + 2z + 2 + z &= 0 \\ x + 3z &= -2 \end{aligned}$$

$$\begin{aligned} x + (-2z - 2) - z &= 2 \\ x - 2z - 2 - z &= 2 \\ x - 3z &= 4 \end{aligned}$$

3. From the last step you should have two new equations with only two variables. Treat these like their own "mini" system of equations and solve for those variables.

$$\begin{aligned} x + 3z &= -2 \\ x - 3z &= 4 \end{aligned}$$

Solve for x in Equation 2:

$$\begin{aligned} x - 3z &= 4 \\ x &= 3z + 4 \end{aligned}$$

Plug x into Equation 1:

$$\begin{aligned} &(3z + 4) + 3z = -2 \\ &6z = -6 \\ &z = -1 \end{aligned}$$

Plug z back in to find x:

$$\begin{aligned} x &= 3z + 4 \\ x &= 3(-1) + 4 \\ x &= -3 + 4 \\ x &= 1 \end{aligned}$$

4. Use the values you've found to plug back in and get the last variable.

$$\begin{aligned} y &= -2z - 2 \\ y &= -2\,(-1) - 2 \\ y &= 2 - 2 \\ y &= 0 \end{aligned}$$

The solution is $\boxed{(1,0,-1).}$

Elimination
Let's solve this system:

$$\begin{aligned} x - y + 2z &= 2 \\ 3x + y + 5z &= 8 \\ 2x - y - 2z &= -7 \end{aligned}$$

1. Create two pairs of equations and eliminate the *same* variable from both pairs.

Equations 1 and 2:

$$\begin{array}{cccccc} x & -\cancel{y} & +2z & = & 2 \\ 3x & +\cancel{y} & +5z & = & 8 \\ \hline 4x & & +7z & = & 10 \end{array}$$

Equations 2 and 3:

$$\begin{array}{cccccc} 3x & +\cancel{y} & +5z & = & 8 \\ 2x & -\cancel{y} & -2z & = & -7 \\ \hline 5x & & +3z & = & 1 \end{array}$$

2. From the last step you should have two new equations with only two variables. Treat these like their own "mini" system of equations and solve for those variables.

$$\begin{aligned} 4x + 7z &= 10 \\ 5x + 3z &= 1 \end{aligned}$$

Get x coefficients to match:

$$\begin{aligned} 5(4x + 7z) &= 5(10) \\ 20x + 35z &= 50 \end{aligned}$$

$$\begin{aligned} -4(5x + 3z) &= -4(1) \\ -20x - 12z &= -4 \end{aligned}$$

Eliminate x:

$$\begin{array}{ccccc} 20x & +35z & = & 50 \\ -20x & -12z & = & -4 \\ \hline & 23z & = & 46 \end{array}$$

$z = 2$

Solve for x:

$$\begin{aligned} 5x + 3z &= 1 \\ 5x + 3(2) &= 1 \\ 5x + 6 &= 1 \\ 5x &= -5 \\ x &= -1 \end{aligned}$$

3. Use the values you've found to plug back in and get the last variable.

$$\begin{aligned} x - y + 2z &= 2 \\ -1 - y + 2(2) &= 2 \\ -y + 3 &= 2 \\ -y &= -1 \\ y &= 1 \end{aligned}$$

The solution is (-1,1,2).

Examples
Solve the following systems of equations:

1.

$$\begin{aligned} -x - y - 2z &= -5 \\ -x + 2y + 7z &= 4 \\ 2x + y + z &= 7 \end{aligned}$$

$(2 + z, 3 - 3z, z)$

2.

$$\begin{aligned} w - y + z &= 2 \\ 2w - 2x + y + z &= 1 \\ -2x + y &= 1 \\ w + y &= -1 \end{aligned}$$

no solution

3.

$$\begin{aligned} 3p + 2r &= 11 \\ q - 7r &= 4 \\ p - 6q &= 1 \end{aligned}$$

$\boxed{\left(4, \frac{1}{2}, -\frac{1}{2}\right)}$

4. Orange juice, an egg sandwich, and a cup of coffee from a local breakfast shop cost a total of \$6.50. The owner posts a notice announcing that, effective the following week, the price of orange juice will increase 25%, and the price of egg sandwiches will increase 20%. After the increase, the same purchase will cost a total of \$7.60, and orange juice will cost \$1 more than coffee. Find the price of each item before the increase.

Orange juice: \$2; egg sandwich: \$3; coffee: \$1.50