Just-In-Time 1-6 Notes

Sections 1-6

1 JIT 1: Real Numbers

Natural Numbers

- $1, 2, 3, 4, 5, 6, \dots$
- These are also sometimes called “counting numbers”.

Whole Numbers

- $0, 1, 2, 3, 4, 5, 6, \dots$
- These are just the natural numbers together with 0.

Integers

- $\dots, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \dots$

Rationals

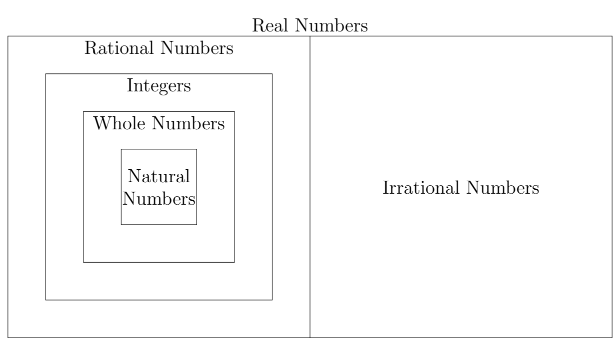
- A rational number is a fraction of two *integers*. For example, $\frac{2}{5}$.
- Every rational number has a decimal expansion that either *terminates* (stops) or *repeats*.
- More examples of rational numbers:

1. $-\frac{17}{8} = \frac{-17}{8}$
2. $2 = \frac{2}{1}$
3. $2.47 = \frac{247}{100}$
4. $0.33333\cdots = \frac{1}{3}$

Irrationals

- An irrational number is any other real number that isn’t a rational number.
- The decimal expansion of irrational numbers is *infinite* and *nonrepeating*.
- Examples of irrational numbers: $\pi \approx 3.14159265\dots$ and $\sqrt{2} \approx 1.41421356\dots$

Real Numbers



Example
Which of the following numbers are rational?

$\frac{\pi}{4}, -\sqrt{16}, 15.75, \frac{1}{\sqrt{17}}, 12$

$$-\sqrt{16} = \frac{-4}{1}$$
$$15.75 = \frac{1575}{100}$$
$$12 = \frac{12}{1}$$

2 JIT 2: Properties of Real Numbers

Commutative Properties

- Commutative Property of Addition:

$$\boxed{x + y = y + x}$$

For example, $5 + 3 = 8$ and $3 + 5 = 8$.

- Commutative Property of Multiplication:

$$\boxed{xy = yx}$$

For example, $3 \cdot 4 = 12$ and $4 \cdot 3 = 12$.

Associative Properties

- Associative Property of Addition:

$$\boxed{x + (y + z) = (x + y) + z}$$

For example, $2 + (3 + (-1)) = 2 + 2 = 4$ and $(2 + 3) + (-1) = 5 - 1 = 4$.

- Associative Property of Multiplication:

$$\boxed{x(yz) = (xy)z}$$

For example, $(2 \cdot 3) \cdot \frac{1}{2} = 6 \cdot \frac{1}{2} = \frac{6}{2} = 3$ and $2 \cdot \left(3 \cdot \frac{1}{2}\right) = 2 \cdot \frac{3}{2} = \frac{6}{2} = 3$.

Identity Properties

- Additive Identity Property:

$$\boxed{0 + x = x + 0 = x}$$

For example, $0 + 3 = 3$.

Zero is the *additive identity.*

- Multiplicative identity property:

$$\boxed{1 \cdot x = x \cdot 1 = x}$$

For example, $-2 \cdot 1 = -2$.

One is the *multiplicative identity.*

Inverse Properties

- Additive Inverse Property:

$$\boxed{-x + x = x + (-x) = 0}$$

For example, $5 + (-5) = 0$.

- Multiplicative Inverse property:

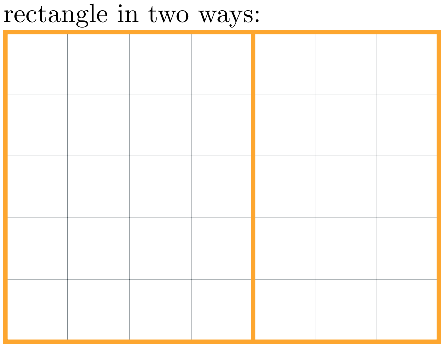
$$\boxed{x \cdot \frac{1}{x} = \frac{1}{x} \cdot x = 1}$$

For example, $3 \cdot \frac{1}{3} = 1$.

Distributive Property

$$\boxed{x(y+z) = xy + xz}$$

For example, $5 \cdot (4+3) = 5 \cdot 7 = 35$ and $5 \cdot 4 + 5 \cdot 3 = 20 + 15 = 35$.
The reasoning behind why this property works can be understood by calculating the area of the big rectangle in two ways:



By multiplying the lengths of the big rectangle:

$$5(7) = 5(4+3)$$

By adding the areas of the two smaller rectangles:

$$20 + 15 = 5(4) + 5(3)$$

Examples
Identify which property is being used.

1. $(5 + (-4)) + 3 = 5 + (-4 + 3)$

   Associative Property of Addition

2. $(ab)c = c(ab)$

   Commutative Property of Multiplication

3. $5 + 0 = 5$

   Additive Identity Property

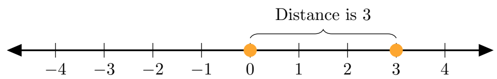
3 JIT 3: Absolute Value

Definition

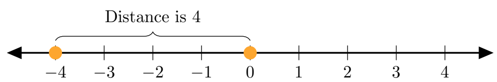
The absolute value is defined algebraically as follows:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Geometrically, it gives the distance between x and 0 on the number line:



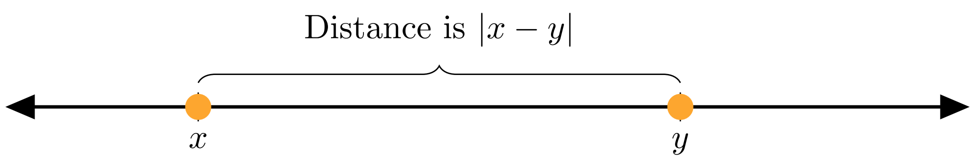
Therefore, $|3| = 3$.



Therefore, $|-4| = 4$.

Distance on the Number Line
If x and y are two numbers on the number line, the distance between them is

$$|x - y|$$



Examples

1. Compute the absolute values: $|-6.2|$ and $\left|\frac{2}{3}\right|$

   Answer: $\boxed{6.2,\ \frac{2}{3}}$

2. Find the distance between 3 and 6.

   Answer: $\boxed{3}$

3. Find the distance between $\frac{12}{5}$ and $\frac{9}{4}$.

   Answer: $\boxed{\frac{3}{20}}$

4 JIT 4: Operations with Real Numbers

Addition

- To add two positive numbers, do normal addition. The answer is positive.

  For example, $3 + 8 = 11$.

- To add a positive number and a negative number, subtract:
  - If the positive number is bigger, the answer is positive.
    For example, $-4 + 16 = 12$
  - If the negative number is bigger, the answer is negative.
    For example, $3 + (-10) = -7$

- To add two negative numbers, add. The answer is negative.

  For example, $-10 + (-3) = -13$.

Subtraction

- We can rewrite subtraction in terms of addition:

$$x - y = x + (-y)$$

  For example, $7 - 15 = 7 + (-15) = -8$.

Multiplication and Division
When you need to multiply or divide two numbers, just figure out the value by multiplying or dividing normally.
The sign of the final answer can be figured out with the following rules:

- If both numbers are positive, the answer is positive.

  For example, $10 \div 5 = 2$.

- If one number is positive and the other is negative, the answer is negative

  For example, $6 \cdot (-3) = -18$, or $(-8) \div 6 = -\frac{8}{6} = -\frac{4}{3}$.

- If both numbers are negative, the answer is positive.

  For example, $(-5) \cdot (-7) = 35$.

5 Fractions

Adding or Subtracting Fractions

1. Get a *common denominator* by multiplying both the top and bottom of each fraction by an appropriate number.
2. Add or subtract the numerators, keeping the common denominator.
3. Simplify the fraction by canceling out common factors from the top and bottom.

Example: $\frac{1}{12} - \frac{5}{28} = -\frac{2}{21}$
Work:

$$12 = 4 \cdot 3$$
$$28 = 4 \cdot 7$$

$$\frac{1}{12} \cdot \frac{7}{7} - \frac{5}{28} \cdot \frac{3}{3} = \frac{7}{84} - \frac{15}{84} = \frac{-8}{84} = -\frac{2}{21}$$

Multiplying Fractions

1. Cancel out common factors. You may cancel factors in either numerator with factors in either denominator.
2. Multiply across the numerators and across the denominators.

Example: $\frac{2}{7} \cdot \frac{21}{8} = \frac{3}{4}$
Work:

$$\frac{2}{7} \cdot \frac{21}{8} = \frac{1}{1} \cdot \frac{3}{4} = \frac{3}{4}$$

Dividing Fractions

1. Change it to a multiplication problem:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad \text{or} \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

2. Finish it just like a multiplication problem.

Example: $\frac{42}{5} \div \frac{35}{2} = \frac{12}{25}$
Work:

$$\frac{42}{5} \div \frac{35}{2} = \frac{42}{5} \cdot \frac{2}{35} = \frac{6}{5} \cdot \frac{2}{5} = \frac{12}{25}$$

Examples
Simplify the following:

1. $\dfrac{-\frac{1}{15}}{\frac{2}{5} - \frac{4}{9}}$

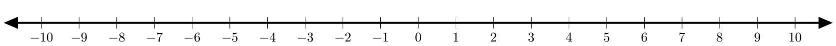
   Answer: $\boxed{\dfrac{3}{2}}$

2. $\dfrac{2 - \frac{10}{11}}{\frac{7}{22} - \frac{3}{4}}$

   Answer: $\boxed{-\frac{48}{19}}$

6 JIT 5: Order on the Number Line

Real Number Line
The real number line is a visual representation of all the real numbers along a line.



$<$, $>$, $\leq$, and $\geq$

- $x < y$ when x is farther left on the number line than y.

  For example:

  - $5 < 6$
  - $-8 < 10$
  - $-7 < -5$

- $x \leq y$ when either $x < y$ or $x = y$.

  For example:

  - $0 \leq 6$
  - $-5 \leq -5$

- $x > y$ when x is farther right on the number line than y.

  For example:

  - $3 > 1$
  - $6 > -4$
  - $-8 > -10$

- $x \geq y$ when either $x > y$ or $x = y$.

  For example:

  - $-2 \geq -5$
  - $3 \geq 3$

Examples
Identify if each is true or false:

1. $-6 < 11$

   True

2. $4 \geq 12$

   False

3. $5 > 5$

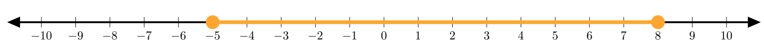
   False

7 JIT 6: Interval Notation

Intervals

An *interval* is a section of the real number line.

Example:



Finite Intervals
A *finite interval* is a section of the number line between two endpoints:

The set of all real numbers between 2 and 3, excluding both 2 and 3.

Algebraic Notatation: $2 < x < 3$
Set Notation: $\{x \mid 2 < x < 3\}$
Interval Notation: $(2, 3)$
Alternative graph:

The set of all real numbers between $\frac{3}{8}$ and 12, including $\frac{3}{8}$ and excluding 12.

Algebraic Notatation: $\frac{3}{8} \leq x < 12$
Set Notation: $\left\{x \mid \frac{3}{8} \leq x < 12\right\}$
Interval Notation: $\left[\frac{3}{8}, 12\right)$
Alternative graph:

The set of all real numbers between -17 and 4, including both -17 and 4.

Algebraic Notatation: $-17 \leq x \leq 4$
Set Notation: $\{x \mid -17 \leq x \leq 4\}$
Interval Notation: $[-17, 4]$
Alternative graph:

The set of all real numbers between -6 and π, excluding -6 and including π.

Algebraic Notatation: $-6 < x \leq \pi$
Set Notation: $\{x \mid -6 < x \leq \pi\}$
Interval Notation: $(-6, \pi]$
Alternative graph:

Infinite Intervals

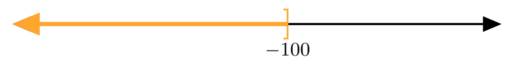
An *infinite interval* is a section of the real number line before or after a single endpoint.

The set of all real numbers smaller than 5, excluding 5.

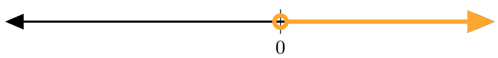
Algebraic Notatation: $x < 5$
Set Notation: $\{x \mid x < 5\}$
Interval Notation: $(-\infty, 5)$
Alternative graph:

The set of all real numbers smaller than -100, including -100.

Algebraic Notatation: $x \leq -100$
Set Notation: $\{x \mid x \leq -100\}$
Interval Notation: $(-\infty, -100]$
Alternative graph:



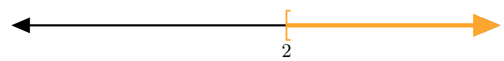
The set of all real numbers bigger than 0, excluding 0.



Algebraic Notatation: $x > 0$
Set Notation: $\{x \mid x > 0\}$
Interval Notation: $(0, \infty)$
Alternative graph:

The set of all real numbers bigger than 2, including 2.

Algebraic Notatation: $x \geq 2$
Set Notation: $\{x \mid x \geq 2\}$
Interval Notation: $[2, \infty)$
Alternative graph:



Examples

1. Graph and write in interval notation:

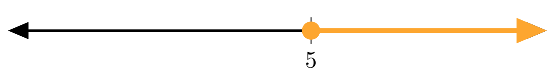
$$-3 < x \leq 7$$

Graph:

or

Interval Notation: $(-3, 7]$

2. Find the algebraic and interval notation for the following graph:



$\boxed{x \geq 5 \text{ and } [5, \infty)}$