Just-In-Time 14-17 Notes

Sections 14-17

1 JIT 14: Equation-Solving Principles

Equation Solving Techniques

1. You can add or subtract any number from both sides of an equation.

$$a = b \text{ means } a + c = b + c$$
$$a = b \text{ means } a - c = b - c$$

2. You can multiply or divide by any nonzero number on both sides of an equation.

$$a = b \text{ means } ac = bc$$
$$a = b \text{ means } \frac{a}{c} = \frac{b}{c}$$

Example
Solve the equation.

1. $11 - 2x = 5x - 3$

   $\boxed{x = 2}$

2. $24 = 5(2t + 5)$

   $\boxed{t = -\frac{1}{10}}$

3. $9x + 3 = 6(2 - x) + 3x$

   $\boxed{x = \frac{3}{4}}$

2 JIT 15: Inequality-Solving Principles

Definition
An *inequality* is a mathematical relation that compares two expressions using "$<$" (less than), "$>$" (greater than), "$\leq$" (less than or equal to), "$\geq$" (greater than or equal to), or "$\neq$" (not equal to). For practical purposes, in this section we'll focus on "$<$", "$>$", "$\leq$", and "$\geq$" because inequalties with "$\neq$" can be solved almost identically to equations (with $=$).

Inquality Solving Techniques

1. You can add or subtract any number from both sides of an inequality.
2. You can multiply or divide both sides of an iequality by any positive number.
3. You can multiply or divide both sides of an inequality by any negative number, *however*, you must then flip the inequality.
   - "$<$" flips with "$>$"
   - "$\leq$" flips with "$\geq$"

   Don't multiply both sides of an inequality by a variable! Since we don't know if the variable is positive or negative, we don't know whether to flip the inequality or not.

Examples
Solve the inequality.

1. $4x < 8x + 7$

   $\boxed{x > -\frac{7}{4}}$ or $\boxed{(-\frac{7}{4}, \infty)}$

2. $\frac{2}{3}y + 6 \geq 8$

   $\boxed{y \geq 3}$ or $\boxed{[3, \infty)}$

3. $-r - 8 > -6$

   $\boxed{r < -2}$ or $\boxed{(-\infty, -2)}$

3 JIT 16: Principle of Zero Products

Solving Equations with Zero on One Side

If several numbers multiply to zero, at least one of those numbers must be zero.

For example:

- $x(x + 3) = 0$ means either $x = 0$ or $x + 3 = 0$. So, either $x = 0$ or $x = -3$.
- $3(x + 2)(x - 1) = 0$ means $3 = 0$, $x + 2 = 0$, or $x - 1 = 0$. Since $3 \neq 0$, the first possibility can't happen, and then we get either $x = -2$ or $x = 1$.

Examples
Solve the equation.

1. $2x^2 - 3x - 5 = 0$

   $\boxed{x = \frac{5}{2},\ x = -1}$

2. $10x^2 + x - 3 = 0$

   $\boxed{x = -\frac{3}{5},\ x = \frac{1}{2}}$

4 JIT 17: Principle of Square Roots

Solving Equations with a Square on One Side

If $x^2 = a$, then $x = \sqrt{a}$ or $x = -\sqrt{a}$ (as long as a isn't negative).

For example, if $x^2 = 9$, then we can either have:

- $3^2 = 3 \cdot 3 = 9$, so $x = 3$, or
- $(-3)^2 = (-3)(-3) = 9$, so $x = -3$.

Examples
Solve the equation.

1. $4y^2 - 100 = 0$

   $\boxed{y = \pm 5}$

2. $3x^2 = 18$

   $\boxed{x = \pm\sqrt{6}}$