

Just-In-Time 18-21 Notes

Sections 18-21

1 JIT 18: Simplifying Rational Expressions

Fractional Expressions

- A *fractional expression* is a quotient of two algebraic expressions.

For example, $\frac{2x}{4x^2-7}$, $\frac{\sqrt{x}-1}{(4-x)^2}$

Rational Expressions

- A *rational expression* is a fractional expression where both the numerator and denominator are polynomials.

For example, $\frac{5x+7}{x^3-x+1}$, $\frac{2}{4t^2-t+3}$

Domain

- Every algebraic expression has a *domain* - the set of all numbers that “make sense” to plug in for your variable.
- Many expressions have the domain “all real numbers.” This means you’re allowed to plug any number you want into the variable.
- However, some operations are undefined when you try to plug in certain numbers, and if the expression uses one of those operations, its domain may have to exclude some numbers.

Domain of Rational Expressions

- How do you compute division by zero? For example, imagine the answer to $\frac{8}{0}$ is the number x .
- Since division is “the opposite” of multiplication, this means that $0 \cdot x = 8$, which simplifies to $0 = 8$. Nonsense!
- When we try to figure out how to divide by zero, we can’t! We say that division by zero is *undefined*.

If the expression contains $\frac{A}{B}$, then $B \neq 0$.

Examples

Find the domain of the following expressions:

1. $\frac{2x+3}{x^2-1}$

$$x \neq \pm 1 \text{ or } (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

2. $\frac{3x+5}{2x-8}$

Answer: $x \neq 4 \text{ or } (-\infty, 4) \cup (4, \infty)$

Simplifying Rational Expressions

To simplify, factor the numerator and denominator and cancel out the common factors.

Examples

Simplify the following:

1. $\frac{8x^3+1}{4x^2-1}$

$$\frac{4x^2+2x+1}{2x-1}$$

2. $\frac{3x^2+x-10}{9x^2-18x+5}$

$$\frac{x+2}{3x-1}$$

2 JIT 19: Multiplying and Dividing Rational Expressions

Method

1. For division problems, change it to a multiplication problem by multiplying the first fraction by the reciprocal of the second.

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} \text{ or } \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

2. Factor all tops and bottoms completely.
3. Cancel common factors (top and bottom of one fraction or diagonally).
4. Multiply across the top and across the bottom - you may leave the answer in factored form.

Examples

Simplify the following:

1. $\frac{x^2-2xy+y^2}{x^2-y^2} \div \frac{2x^2-xy-y^2}{x^2+xy-2y^2}$

$$\frac{(x+2y)(x-y)}{(x+y)(2x+y)}$$

$$2. \frac{6t^2 - t - 1}{2t^2 - 5t + 2} \cdot \frac{t^2 - 4}{3t^2 - 5t - 2}$$

$$\frac{t+2}{t-2}$$

3 JIT 20: Add and Subtract Rational Expressions

Method

1. Factor all denominators completely.
2. Get common denominators.
3. Add or subtract the numerators.
4. Factor the numerator and simplify by cancelling on top and bottom if possible.

Examples

Simplify the following:

$$1. \frac{x+2}{x^2+x-2} + \frac{x+2}{x^2-5x+4}$$

Answer: $\frac{2}{x-4}$

$$2. \frac{2}{x} + \frac{3}{x-1} - \frac{4}{x^2-x}$$

Answer: $\frac{5x-6}{x(x-1)}$

4 JIT 21: Simplify Complex Rational Expressions

Definition

- A *complex fraction*, also called a *compound fraction*, is a fraction where the numerator, the denominator, or both contain fractions.
- For example, $\frac{x+5}{\frac{1}{x}+3}$ and $\frac{\frac{x+2}{x} - \frac{x+3}{x^2}}{2 - \frac{x+5}{x}}$ are both complex fractions.

Method 1

1. Simplify the top and bottom each into a single fraction.
2. Multiply the top fraction by the reciprocal of the bottom fraction.

Method 2

1. Factor all the denominators of the “small” fractions and determine the least common multiply (LCM) of them.
2. Multiply the top and bottom of the “big” fraction by the LCM to cancel out all the “small” denominators.
3. Simplify.

Examples

Simplify into a single fraction:

1. $\frac{\frac{1}{a^2} - \frac{1}{b^2}}{a - b}$

$$-\frac{a + b}{a^2b^2}$$

2. $\frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{y} + \frac{1}{x}}$

$$x - y$$