Just-In-Time 22-25 Notes

Sections 22-25

1 JIT 22: Simplify Radical Expressions

Definition of nth Roots

- If n is a natural number and $y^n = x$, then $\sqrt[n]{x} = y$.
- $\sqrt[n]{x}$ is read as "the nth root of x".
  - $\sqrt[3]{8} = 2$ because $2^3 = 8$.
  - $\sqrt[4]{81} = 3$ because $3^4 = 81$.
- When $n = 2$ we call it a "square root". However, instead of writing $\sqrt[2]{x}$, we drop the 2 and just write $\sqrt{x}$. So, for example:
  - $\sqrt{16} = 4$ because $4^2 = 16$.

Domain of Radical Expressions

- How do you find the even root of a negative number? For example, imagine the answer to $\sqrt[4]{-16}$ is the number x.

  This means $x^4 = -16$. The problem is, if we raise a number to the fourth power, we *never* get a negative answer:

$$(2)^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$
$$(-2)^4 = (-2)(-2)(-2)(-2) = 16$$

- Since there is no solution here for x, we say that the fourth root is undefined for negative numbers.
- However, the same idea applies to *any* even root (square roots, fourth roots, sixth roots, etc)

> If the expression contains $\boxed{\sqrt[n]{B}}$ where n is even, then $\boxed{B \geq 0}$.

Properties of Roots/Radicals

- $\sqrt[n]{x^n} = x$ if n is odd.
- $\sqrt[n]{x^n} = |x|$ if n is even.
- $\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$
  (When n is even, x and y need to be nonnegative.)
- $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$
  (When n is even, x and y need to be nonnegative.)
- $\sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$
  (When n is even, x needs to be nonnegative.)
- $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$

Examples

1. $\sqrt[3]{375} + \sqrt[3]{-81}$

   $\boxed{2\sqrt[3]{3}}$

2. $\sqrt{16x^2}$

   $\boxed{4|x|}$

3. $\sqrt[5]{x^3y^4}\,\sqrt[5]{x^7y}$

   $\boxed{x^2y}$

4. $\sqrt[3]{\frac{16ab}{a^4b^3}}$

   $\boxed{\frac{2}{a}\sqrt[3]{\frac{2}{b^2}}}$

2 JIT 23: Rationalizing Numerators and Denominators

Definition

- *Rationalizing the denominator* of a fraction is simplifying the fraction so that the denominator doesn't have any roots or radicals.
- *Rationalizing the numerator* of a fraction is simplifying the fraction so that the numerator doesn't have any roots or radicals.

Methods

The numerator/denominator has *no* addition or subtraction:

- Simplify any roots as much as possible.
- The numerator/ denominator you're trying to rationalize should have $\sqrt[n]{x^m}$.
- Multiply the top and bottom of the fraction by $\sqrt[n]{x^{n-m}}$.

The numerator/denominator has addition or subtraction:

- To rationalize, multiply the top and bottom of the fraction by the *conjugate* - the expression you get when you flip the sign in the "middle".

  $a\sqrt{x} + b\sqrt{y} \longleftrightarrow a\sqrt{x} - b\sqrt{y}$

Examples

1. Rationalize the denominator:

$$\frac{3}{2-\sqrt{7}}$$

$\boxed{-2-\sqrt{7}}$

2. Rationalize the numerator:

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-4\sqrt{2}}$$

$\boxed{\dfrac{1}{11-5\sqrt{6}}}$

3. Rationalize the denominator:

$$\sqrt[3]{\frac{3}{5}}$$

$\boxed{\dfrac{\sqrt[3]{75}}{5}}$

4. Rationalize the numerator:

$$\frac{\sqrt{7}}{2}$$

$\boxed{\dfrac{7}{2\sqrt{7}}}$

3 JIT 24: Rational Exponents

Definition of Rational Exponents

Every nth root has an equivalent exponential form:

$$\sqrt[n]{x} = x^{1/n}$$

When roots are written in their exponential form, you can use all of the properties of exponents to simplify problems.

Another nice formula to convert is

$$\sqrt[n]{x^m} = x^{m/n}$$

Examples

Simplify the following expressions, writing your final answer without negative exponents. Assume all variables denote positive quantities.

1. $\sqrt[3]{a^4b^{-1}}\sqrt[9]{a^6b^2}$

   $\boxed{\dfrac{a^2}{b^{1/9}}}$

2. $\left(\dfrac{r^8s^{-4}}{16s^{4/3}}\right)^{-1/4}$

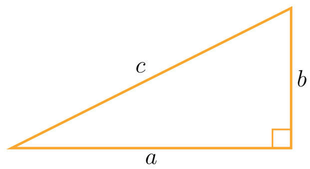
   $\boxed{\dfrac{2s^{4/3}}{r^2}}$

4 JIT 25: Pythagorean Theorem

Formula

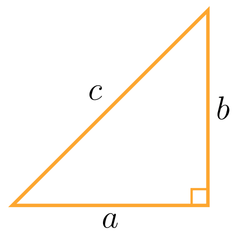
The Pythagorean Theorem is a formula that relates the lengths of the sides of a *right triangle* (a triangle where one of the angles is 90°).

$$\boxed{a^2 + b^2 = c^2}$$



Examples

Find the length of the side not given.



1. $a = 4$, $b = 6$

   $\boxed{2\sqrt{13}}$

2. $a = 3$, $c = 5$

   $\boxed{4}$