

Just-In-Time 7-9 Notes

Sections 7-9

1 JIT 7: Integers as Exponents

Natural Number Exponents

- If n is a natural number,

$$x^n = \underbrace{x \cdot x \cdot x \dots x}_{n \text{ times}}$$

- When we use this notation, x is called the *base* and n is called the *exponent*.
- Examples:

1. $9^2 = 9 \cdot 9 = 81$

2. $2^3 = 2 \cdot 2 \cdot 2 = 4 \cdot 2 = 8$

3. $(a+2)^5 = (a+2)(a+2)(a+2)(a+2)(a+2)$

Integer Exponents

- If m is an integer, $x^{-m} = \frac{1}{x^m}$ (and also $\frac{1}{x^{-m}} = x^m$).
- Examples:

1. $2^{-4} = \frac{1}{2^4} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{16}$

2. $\frac{1}{5^{-3}} = 5^3 = 125$

Properties of Exponents

1. $x^a \cdot x^b = x^{a+b}$

$$2^4 \cdot 2^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$$

2. $\frac{x^a}{x^b} = x^{a-b}$

$$\frac{4^5}{4^3} = \frac{4 \cdot 4 \cdot \cancel{4} \cdot \cancel{4} \cdot \cancel{4}}{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}} = 4^2$$

3. $(x^a)^b = x^{ab}$

$$(5^4)^3 = 5^4 \cdot 5^4 \cdot 5^4 = 5^{4+4+4} = 5^{12}$$

4. $(xy)^a = x^a y^a$

$$(2z)^3 = 2z \cdot 2z \cdot 2z = 2 \cdot 2 \cdot 2 \cdot z \cdot z \cdot z = 2^3 z^3$$

5. $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

$$\left(\frac{7}{2}\right)^4 = \frac{7}{2} \cdot \frac{7}{2} \cdot \frac{7}{2} \cdot \frac{7}{2} = \frac{7 \cdot 7 \cdot 7 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{7^4}{2^4}$$

$$6. \left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$$

$$\left(\frac{4}{3}\right)^{-2} = \frac{4^{-2}}{3^{-2}} = \frac{\frac{1}{4^2}}{\frac{1}{3^2}} = \frac{1}{4^2} \cdot \frac{3^2}{1} = \frac{3^2}{4^2}$$

$$7. \frac{x^{-a}}{y^{-b}} = \frac{y^b}{x^a}$$

$$\frac{5^{-3}}{2^{-4}} = \frac{\frac{1}{5^3}}{\frac{1}{2^4}} = \frac{1}{5^3} \cdot \frac{2^4}{1} = \frac{2^4}{5^3}$$

Special Cases

- $x^1 = x$
- $x^0 = 1$ as long as $x \neq 0$. (0^0 is undefined.)

Examples

Simplify the following expressions and eliminate any negative exponents.

$$1. \frac{(3x)^3 y}{xy^4}$$

$$\frac{27x^2}{y^3}$$

$$2. \frac{3x^2 y^{-4}}{9x^3 y^{-10}}$$

$$\frac{y^6}{3x}$$

$$3. \left(\frac{2a^2}{b}\right)^4 \left(\frac{b^2}{4a^3}\right)^2$$

$$a^2$$

$$4. \left(\frac{2q^{-4}r^{-3}s}{3r^4s^{-3}}\right)^{-2}$$

$$\frac{9r^{14}q^8}{4s^8}$$

2 JIT 8: Scientific Notation

Definition of Scientific Notation

A number is in scientific notation if it is written in the form

$$a \times 10^n$$

where a has exactly one non-zero digit left of the decimal point, and n is an integer.
For example: 2.37×10^6 and -1.0021×10^{-100} .

Examples

Convert the following numbers from scientific notation to decimal notation:

1. -3.001×10^5

-300,100

2. 2.42×10^{-3}

0.00242

Examples

Convert the following numbers from decimal notation to scientific notation:

1. 25,040,000

2.504×10^7

2. 1.3

1.3×10^0

3. -0.09624

-9.624×10^{-2}

3 JIT 9: Order of Operations

Rules

1. Simplify inside grouping symbols first (for example: parenthesis, brackets, inside square roots, top and bottom of fractions).
2. Evaluate exponents.
3. Perform all multiplications and divisions, left to right.
4. Perform all addition and subtraction, left to right.

The acronym PEMDAS is a common device to help remember the order: parenthesis, exponents, multiplication/division, addition/subtraction.

Examples

Simplify each of the following:

1. $3 \cdot 15 - 4 \cdot 2^4 + 2(1 - 7)$

-31

2. $4^2 \cdot 81 \div 3^3 \cdot 2^{-3}$

6

3. $\frac{6^2 - 2(3 - 5)^2}{4^3 - 32}$

$\frac{7}{8}$