

Section 1.10 Notes

Lines

Definitions

- Every *line* has the equation

$$Ax + By = C$$

where at least one of A or B is not zero.

- Every non-vertical line has a *slope*, a number which describes the steepness of the line. If (x_1, y_1) and (x_2, y_2) are points on the line:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Forms of Lines

- Slope-Intercept Form of a Line:

$$y = mx + b$$

where m is the slope and b is the y -intercept

- Point-Slope Form of a Line:

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is a point on the line

Special Types of Lines

- Horizontal Line:

$$y = b$$

where b is the y -intercept

Horizontal lines have a slope of zero.

- Vertical Line:

$$x = a$$

where a is the x -intercept

Vertical lines have no slope - in other words, their slope is undefined.

Examples

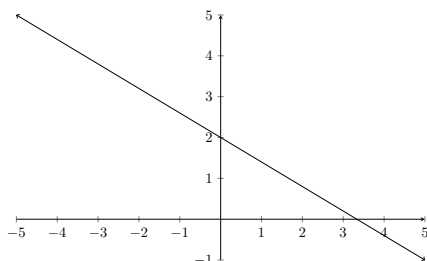
1. Find the equation of the line which passes through $(1,2)$ and $(3,3)$.

$$y = \frac{1}{2}x + \frac{5}{2}$$

2. Find the slope, y -intercept and graph the line: $3x + 5y = 10$

slope: $-\frac{3}{5}$

y-intercept: 2



Parallel and Perpendicular Lines

1. Parallel lines:

- Two lines with defined slope are parallel when they have the same slope.

$$m_1 = m_2$$

- Two vertical lines are parallel.

2. Perpendicular lines:

- Two lines with defined slope are perpendicular when they have negative reciprocal slopes.

$$m_1 = -\frac{1}{m_2}$$

- Vertical lines and horizontal lines are perpendicular.

Examples

1. Find the line which passes through (2,1) and is parallel to $x + 3y = 6$.

$$y - 1 = -\frac{1}{3}(x - 2)$$

2. Find the line which passes through $(-\frac{1}{2}, 4)$ and is perpendicular to $y = -3$.

$$x = -\frac{1}{2}$$

3. Find the line which passes through (1,-1) and is perpendicular to $y - 2x = 3$.

$$y + 1 = -\frac{1}{2}(x - 1)$$