Section 1.4 Notes

Rational Expressions

1 Definitions

Fractional Expressions

- A *fractional expression* is a quotient of two algebraic expressions.

  For example, $\frac{2x}{4x^2-7}$, $\frac{\sqrt{x}-1}{(4-x)^2}$

Rational Expressions

- A *rational expression* is a fractional expression where both the numerator and denominator are polynomials.

  For example, $\frac{5x+7}{x^3-x+1}$, $\frac{2}{4t^2-t+3}$

2 Domain

Definition

- Every algebraic expression has a *domain* - the set of all numbers that "make sense" to plug in for your variable.
- Many expressions have the domain "all real numbers." This means you're allowed to plug any number you want into the variable.
- However, some operations are undefined when you try to plug in certain numbers, and if the expression uses one of those operations, its domain may have to exclude some numbers.

Division by Zero

- How do you compute division by zero? For example, imagine the answer to $\frac{8}{0}$ is the number x.
- Since divsion is "the opposite" of multiplication, this means that $0 \cdot x = 8$, which simplifies to $0 = 8$. Nonsense!
- When we try to figure out how to divide by zero, we can't! We say that division by zero is *undefined*.

Square Root of a Negative

- How do you find the square root of a negative number? For example, imagine the answer to $\sqrt{-9}$ is the number x.

  This means $x^2 = -9$. The problem is, if we square a number, we *never* get a negative answer:

$$(3)^2 = 9$$
$$(-3)^2 = (-3)(-3) = 9$$

- Since there is no solution here for x, we say that the square root is undefined for negative numbers.

Rules for Finding Domain

1. If the expression contains $\boxed{\dfrac{A}{B}}$, then $\boxed{B \neq 0}$.
2. If the expression contains $\boxed{\sqrt{B}}$, then $\boxed{B \geq 0}$.

- If the expression contains $\boxed{\dfrac{A}{\sqrt{B}}}$, then $\boxed{B > 0}$. This rule *only* works when the *entire* denominator is under a square root.

Examples
Find the domain of the following expressions:

1. $\dfrac{2x+3}{x^2-1}$

   Answer: $\boxed{(-\infty,-1)\cup(-1,1)\cup(1,\infty)}$

2. $\dfrac{\sqrt{x-3}}{2x-8}$

   Answer: $\boxed{[3,4)\cup(4,\infty)}$

3. $\dfrac{2+x}{\sqrt{5-x}}$

   Answer: $\boxed{(-\infty,5)}$

3 Working with Rational Expressions

Simplifying Rational Expressions
To simplify, factor the numerator and denominator and cancel out the common factors.

Examples
Simplify the following:

1. $\dfrac{8x^3+1}{4x^2-1}$

   $\boxed{\dfrac{4x^2+2x+1}{2x-1}}$

2. $\dfrac{3x^2+x-10}{9x^2-18x+5}$

   $\boxed{\dfrac{x+2}{3x-1}}$

Multiplying and Dividing Rational Expressions

1. For division problems, change it to a multiplication problem by multiplying the first fraction by the reciprocal of the second.
2. Factor all tops and bottoms completely.
3. Cancel common factors (top and botton of one fraction or diagonally).
4. Multiply across the top and across the bottom - you may leave the answer in factored form.

Examples
Simplify the following:

1. $\dfrac{x^2-2xy+y^2}{x^2-y^2} \div \dfrac{2x^2-xy-y^2}{x^2+xy-2y^2}$

   Answer: $\boxed{\dfrac{(x+2y)(x-y)}{(x+y)(2x+y)}}$

2. $\dfrac{6t^2-t-1}{2t^2-5t+2} \cdot \dfrac{t^2-4}{3t^2-5t-2}$

   Answer: $\boxed{\dfrac{t+2}{t-2}}$

Adding and Subtracting Rational Expressions

1. Factor all denominators completely.
2. Get common denominators.
3. Add or subtract the numerators.
4. Factor the numerator and simplify by cancelling on top and bottom if possible.

Examples
Simplify the following:

1. $\dfrac{x+2}{x^2+x-2} + \dfrac{x+2}{x^2-5x+4}$

   Answer: $\boxed{\dfrac{2}{x-4}}$

2. $\dfrac{2}{x} + \dfrac{3}{x-1} - \dfrac{4}{x^2-x}$

   Answer: $\boxed{\dfrac{5x-6}{x(x-1)}}$

4 Compound Fractions

Definition

- A *compound fraction* is a fraction where the numerator, the denominator, or both contain fractions.
- For example, $\dfrac{x+5}{\frac{1}{x}+3}$ and $\dfrac{\frac{x+2}{x}-\frac{x+3}{x^2}}{2-\frac{x+5}{x}}$ are both compound fractions.

Method 1

1. Simplify the top and bottom each into a single fraction.
2. Multiply the top fraction by the reciprocal of the bottom fraction.

Method 2

1. Factor all the denominators of the "small" fractions and determine the least common multiply (LCM) of them.
2. Multiply the top and bottom of the "big" fracion by the LCM to cancel out all the "small" denominators.
3. Simplify.

Examples
Simplify into a single fraction:

1. $\dfrac{\frac{1}{a^2}-\frac{1}{b^2}}{a-b}$

$$\boxed{-\frac{a+b}{a^2b^2}}$$

2. $\dfrac{\frac{s+1}{s-1}-\frac{s+2}{s-2}}{\frac{2s-3}{s^2-3s+2}+\frac{s-2}{s-1}}$

$$\boxed{\frac{-2s}{(s-1)^2}}$$

5 Rationalizing Numerators and Denominators

Definition

- *Rationalizing the denominator* of a fraction is simplifying the fraction so that the denominator doesn't have any roots or radicals.
- *Rationalizing the numerator* of a fraction is simplifying the fraction so that the numerator doesn't have any roots or radicals.
- To rationalize, multiply the top and bottom of the fraction by the *conjugate* - the expression you get when you flip the sign in the "middle".

$$a\sqrt{x}+b\sqrt{y}\longleftrightarrow a\sqrt{x}-b\sqrt{y}$$

Examples

1. Rationalize the denominator:

$$\frac{3}{2-\sqrt{x}}$$

$$\boxed{\frac{3(2+\sqrt{x})}{4-x}}$$

2. Rationalize the numerator:

$$\frac{\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{y}}}{y-x}$$

$$\boxed{\frac{1}{\sqrt{xy}(\sqrt{y}+\sqrt{x})}}$$