

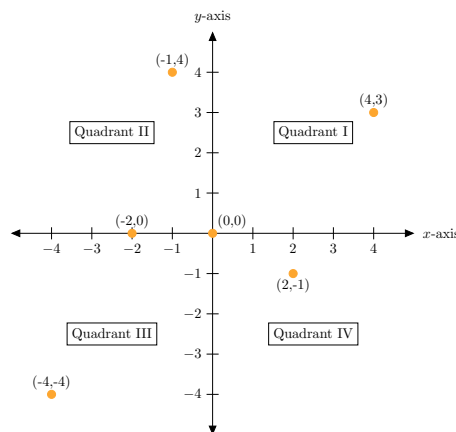
Section 1.8 Notes

Coordinate Geometry

1 Introduction

The Coordinate Plane

- The *Cartesian* or *coordinate plane* is a two-dimensional plane where the points are described using a pair of numbers (x, y) . The first number gives the horizontal location and the second number gives the vertical location of the point.
- The *origin* is the name for the “center” at $(0,0)$.

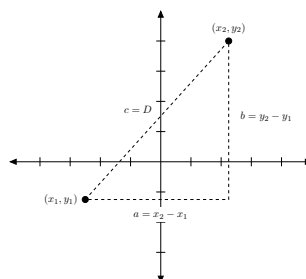


2 Formulas

Distance Formula

The distance between (x_1, y_1) and (x_2, y_2) on the plane is given by the formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$$a^2 + b^2 = c^2 \text{ (Pythagorean Theorem)}$$

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = D^2$$

$$\pm \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{D^2}$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = D \text{ (distance is always positive)}$$

Example

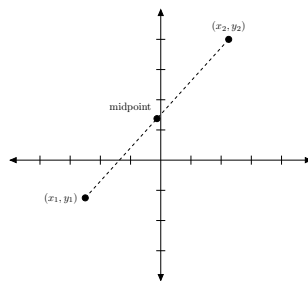
Find the distance between $(2, -4)$ and $(-1, -1)$.

$$3\sqrt{2}$$

Midpoint Formula

The midpoint between (x_1, y_1) and (x_2, y_2) on the plane is given by the formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Examples

1. Find the midpoint between $(\frac{1}{3}, -2)$ and $(\frac{4}{3}, -3)$.

$$\left(\frac{5}{6}, -\frac{5}{2} \right)$$

2. If A has coordinates $(1, 2)$ and the midpoint between A and B is $(-3, 1)$, find B .

$$(-7, 0)$$

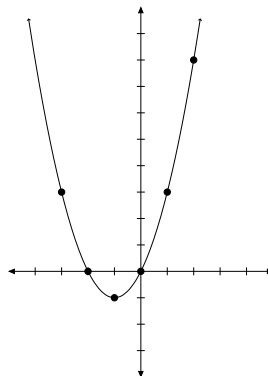
3 Graphs and their Features

Graphing Equations with Two Variables

The *graph* of an equation is the visual representation of all the ordered pairs which satisfy the equation - in other words, if the x -value is plugged in to x , and the y -value is plugged into y , you get a true statement.

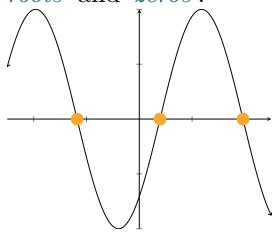
Example: Graph $y = x^2 + 2x$.

x	y
-3	$(-3)^2 + 2 \cdot (-3) = 3$
-2	$(-2)^2 + 2 \cdot (-2) = 0$
-1	$(-1)^2 + 2 \cdot (-1) = -1$
0	$0^2 + 2 \cdot 0 = 0$
1	$1^2 + 2 \cdot 1 = 3$
2	$2^2 + 2 \cdot 2 = 8$



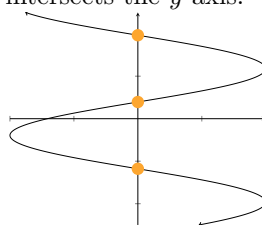
Intercepts

An *x-intercept* is a point where the graph intersects the *x*-axis. These are also called *roots* and *zeros*.



To find algebraically, set $y = 0$ and solve.

A *y-intercept* is a point where the graph intersects the *y*-axis.



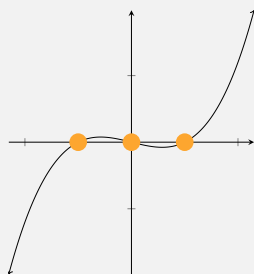
To find algebraically, set $x = 0$ and solve.

Example

Find the intercepts of $y = x^3 - x$ and graph the equation.

x-intercepts: $(0,0)$, $(1,0)$, $(-1,0)$

y-intercept: $(0,0)$



Symmetry

A graph has *symmetry across the x-axis* when it looks the same after flipping across the *x*-axis.

In the equation, if you replace y by $-y$ and simplify, the equation will be unchanged.

A graph has *symmetry about the origin* when it looks the same after rotating 180° around the origin.

In the equation, if you replace x by $-x$ and y by $-y$ and simplify, the equation will be unchanged.

A graph has *symmetry across the y-axis* when it looks the same after flipping across the *y*-axis.

In the equation, if you replace x by $-x$ and simplify, the equation will be unchanged.

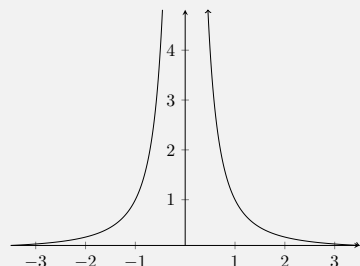
Example

Check for symmetry and graph the equation $x^2y = 2$.

Symmetry across x -axis: no

Symmetry across y -axis: yes

Symmetry about the origin: no

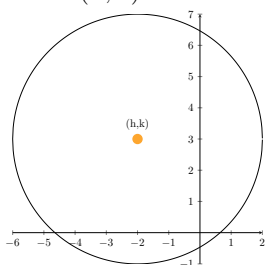


Circles

The equation of a circle is given by

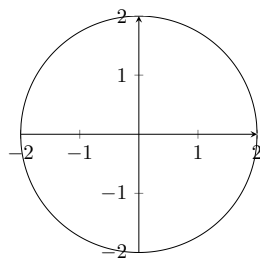
$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the center of the circle and $r > 0$ is the radius.

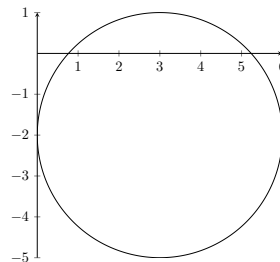


Examples

1. Graph $x^2 + y^2 = 4$.



2. Graph $(x - 3)^2 + (y + 2) = 9$.



1. Find the equation of the circle in the second quadrant with radius 2 which is tangent to both the x and y -axes.

$$(x + 2)^2 + (y - 2)^2 = 4$$

2. Show the equation is a circle and find the center and radius of the circle: $x^2 + y^2 + 6x - 2y - 6 = 0$.

$$(x + 3)^2 + (y - 1)^2 = 16$$

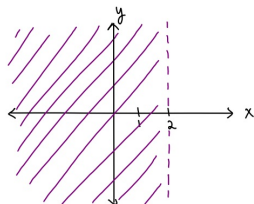
Center: $(-3, 1)$
Radius: 4

Graphing Inequalities on the Plane

- Graph the lines you get when replacing the inequality with an equal sign.
 - $<$ or $>$ - dashed lines
 - $\leq$ or $\geq$ - solid lines
- The plane will be cut up into different regions. Pick test points from each region and plug in to the inequality to see where to shade.

Examples

- Graph $\{(x, y) \mid x < 2\}$



- Graph $\{(x, y) \mid |y - 2| \leq 1\}$

