Section 10.1 Notes

Systems of Linear Equations in Two Variable

1 Introduction

Definitions

- A *system of equations* is a list of two or more equations.
- A *linear system of equations* has only linear equations in the list.

  For example:

$$\begin{aligned} 2x - y &= 4 \\ x + y &= 2 \end{aligned}$$

- A *solution* to a system of equations is a pair of x and y-values that, when plugged in, make all the equations true.
- If a system has more than two variables, every solution consists of a number assigned to each variable.

  For example, the solution to the above system is $(2, 0)$ or $x = 2$, $y = 2$ because:

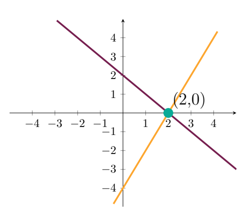
$$\begin{aligned} 2(2) - 0 &= 4 \checkmark \\ 2 + 0 &= 2 \checkmark \end{aligned}$$

Graphing

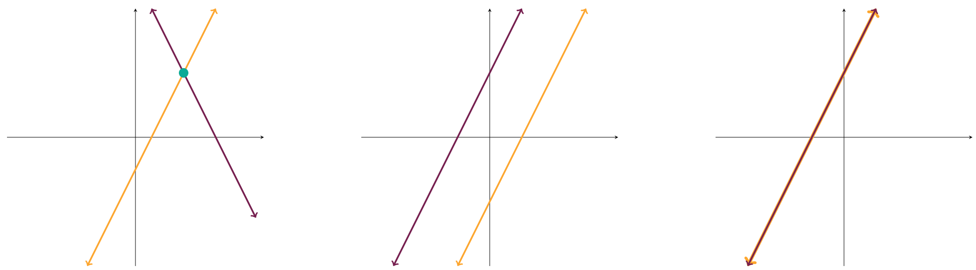
- We can graph the equations in a system on a single coordinate plane.
- When we do this, the point(s) where the lines/curves cross is/are the solution(s).

  For example:

$$\begin{aligned} 2x - y = 4 &\to -y = -2x + 4 \to \boxed{y = 2x - 4} \\ x + y = 2 &\to \boxed{y = -x + 2} \end{aligned}$$



The Number of Solutions of a Linear System
There are three possible cases for the number of solutions a linear system of two variables and two equations has:



The two lines cross at a single point.
One solution.

The two lines are parallel and never cross.
No solutions.

The two equations actually represent the same line.
Infinitely many solutions.

- A system is *consistent* if it has at least one solution.
- A system is *inconsistent* if it has no solutions.
- A linear system with two equations is *dependent* when one of the equations simplifies to the other.
- A linear system with two equations is *independent* when it's not dependent.

2 Algebraic Methods of Solving Systems

Substitution
Let's solve this system:

$$\begin{aligned} x - y &= 1 \\ 4x + 3y &= 18 \end{aligned}$$

1. Pick one of the two equations, and solve for either of the variables in this equation.

$$\text{Equation 1: } x = y + 1$$

2. Plug this back into the other equation and solve.

$$\begin{aligned} 4x + 3y &= 18 \\ 4(y+1) + 3y &= 18 \\ 4y + 4 + 3y &= 18 \\ 7y &= 14 \\ y &= 2 \end{aligned}$$

3. Plug back in to any of the equations and solve for the final variable.

$$\begin{aligned} x &= y + 1 \\ x &= 2 + 1 \\ x &= 3 \end{aligned}$$

The solution is $\boxed{(3,2).}$

4. In step 2, if you didn't get $x =\#$ or $y =\#$:
   - The equation simplified to something true (3=3). In this case you have infinitely many solutions. You need to write a formula for every possible solution as an ordered pair.
   - The equation simplified to something false (1=7). In this case you have no solutions.

Elimination
Let's solve this system:

$$\begin{aligned} 3x - 5y &= -11 \\ 4x + 2y &= -6 \end{aligned}$$

1. Pick a variable to eliminate.

   We'll get rid of y.

2. Multiply the equations by appropriate numbers to get the coefficient of your chosen variable to match.

$$\begin{aligned} 2(3x - 5y) = 2(-11) &\rightarrow 6x - \boxed{10}\,y = -22 \\ 5(4x + 2y) = 5(-6) &\rightarrow 20x + \boxed{10}\,y = -30 \end{aligned}$$

3. Add or subtract the two equations and solve (add if the signs on the coefficients are opposite, subtract if they're the same).

$$\begin{array}{llcl} 6x & -\cancel{10\text{y}} & = & -22 \\ 20x & +\cancel{10\text{y}} & = & -30 \\ \hline 26x & & = & -52 \end{array}$$

$$x = -2$$

4. Plug back in to any of the equations and solve for the final variable.

$$\begin{aligned} 3x - 5y &= -11 \\ 3(-2) - 5y &= -11 \\ -6 - 5y &= -11 \\ -5y &= -5 \\ y &= 1 \end{aligned}$$

   The solution is $\boxed{\text{(-2,1)}}$.

5. Again, in step 3, if you didn't get $x =\#$ or $y =\#$:
   - The equation simplified to something true (3=3). In this case you have infinitely many solutions. You need to write a formula for every possible solution as an ordered pair.
   - The equation simplified to something false (1=7). In this case you have no solutions.

Examples

1.

$$\begin{aligned} 5x + 10y &= 7 \\ x + 2y &= -3 \end{aligned}$$

$\boxed{\text{No solutions}}$

2.

$$\begin{aligned} -4x + 3y &= 0 \\ 3x + 4y &= \frac{25}{4} \end{aligned}$$

$\boxed{\left(\frac{3}{4}, 1\right)}$

3.

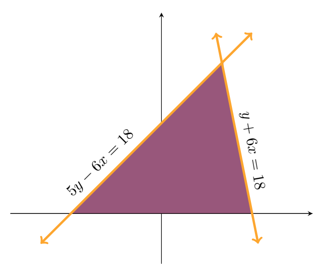
$$\begin{aligned} 2x - y &= 6 \\ 6x &= 3y + 18 \end{aligned}$$

$\boxed{(x, 2x - 6) \text{ or } \left(\frac{1}{2}y + 3, y\right)}$

4. Find two numbers whose sum is 34 and whose difference is 10.

$\boxed{22 \text{ and } 12}$

5. Find the area of the triangle:



$\boxed{18}$