Section 2.1 Notes

What is a Function?

1 Introduction

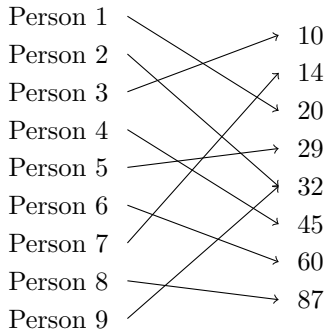
Definitions

- A *function* is a rule that assigns inputs to outputs. Each input must have exactly one output - in other words, no single input can have two or more outputs.
- The *domain* of a function is the set of all inputs.
- The *range* of a function is the set of all outputs.

Functions versus General "Rules"

Some Made-Up People:
Person 1: Noah Smith, 20 years old
Person 2: Emma Johnson, 32 years old
Person 3: Liam Brown, 10 years old
Person 4: Olivia Miller, 45 years old
Person 5: William Davis, 29 years old
Person 6: Ava Garcia, 60 years old
Person 7: Mason Rodriguez, 14 years old
Person 8: Sophia Wilson, 87 years old
Person 9: Willam Anderson, 32 years old

Rule 1: For each person, output their age:



Rule 1 *is* a function.

Some Made-Up People:
Person 1: Noah Smith, 20 years old
Person 2: Emma Johnson, 32 years old
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Person 7: Mason Rodriguez, 14 years old
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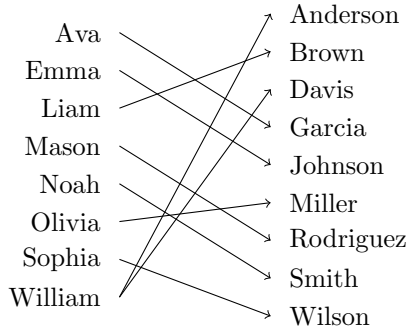
Rule 2: For each first name, output their last name:



Rule 2 *is not* a function because of the first name "William".

Function Notation

- In this class, the functions we will be dealing with will be algebraic. The inputs will be numbers, the outputs will be numbers, and the rule describing how to get the outputs will be an algebraic formula.

- We use a special notation to write functions:

$$\underbrace{f(x)}_{\text{function name}} \underbrace{=}_{\text{input variable}} \underbrace{2x+4}_{\text{formula}}$$

- "$f(x)$" is read as "f of x".
- $f(a)$ is the function f evaluated at the point $x = a$.

Examples

1. If $f(x) = \frac{2x-7}{x^2+1}$, find $f(3)$.

   $\boxed{-\frac{1}{10}}$

2. Express the rule in function notation: "Take the cube root, subtract 3, then multiply by 7".

   $\boxed{f(x) = 7(\sqrt[3]{x} - 3)}$

3. If $f(x) = x^3 - 2$, find $f(a+2)$.

   $\boxed{a^3 + 6a^2 + 12a + 6}$

4. If $f(x) = 2x^2 + 3x$, find $\frac{f(a+h)-f(a)}{h}$.

   $\boxed{4a + h + 3}$

Finding the Domain of a Function

Finding the domain of a function works exactly like finding the domain of an expression:

1. If the expression contains $\boxed{\frac{A}{B}}$, then $\boxed{B \neq 0}$.
2. If the expression contains $\boxed{\sqrt{B}}$, then $\boxed{B \geq 0}$.

- If the expression contains $\boxed{\frac{A}{\sqrt{B}}}$, then $\boxed{B > 0}$. This rule *only* works when the *entire* denominator is under a square root.

Examples

Find the domains in interval notation:

1. $f(x) = x^3 - 3x + 1$

   $\boxed{(-\infty, \infty)}$

2. $g(x) = \frac{1}{x+1}$

$(-\infty, -1) \cup (-1, \infty)$

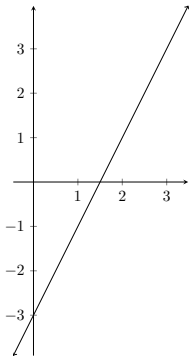
3. $h(x) = \sqrt{3x^2 - 12}$

$(-\infty, -2] \cup [2, \infty)$

2 Graphs of Functions

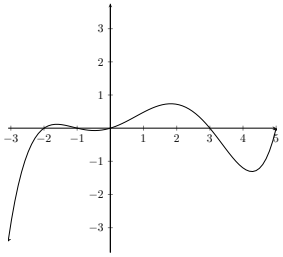
Process

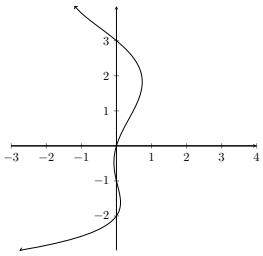
With functions, x is the input and $f(x)$ is the output. To graph, we can treat $f(x)$ just like y. For example, graphing $f(x) = 2x - 3$ is exactly like graphing $y = 2x - 3$:

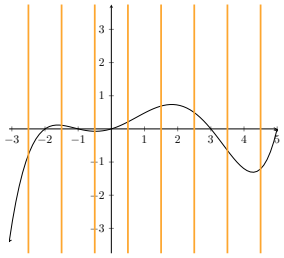


The Vertical Line Test

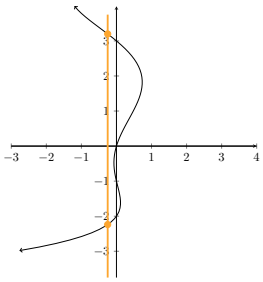
- Any equation with x and y can be graphed, but not all of them correspond to functions (with x as the input and y as the output).
- To be a function, each input can have just *one* output.
- *Vertical Line Test*: A graph passes the Vertical Line Test if every vertical line intersects the graph at most once.
- If the graph of an equation passes the Vertical Line Test, it *is* a function. If it fails, it's not.







This is a function.



This is not a function.

3 Piecewise Functions

Definition

A *piecewise function* has several formulas to compute the output. The formula used depends on the input value.

For example,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Examples

If $h(t) = \begin{cases} 0 & \text{if } t < -2 \\ \frac{12t}{t-1} & \text{if } -2 \leq t < 1 \\ 4t - 3 & \text{if } t \geq 1 \end{cases}$, find

1. $h(0)$

   0

2. $h\left(\frac{4}{3}\right)$

   $\frac{7}{3}$

3. $h(-100)$

   0