Section 2.5 Notes

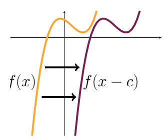
Transformations of Functions

1 Transformations

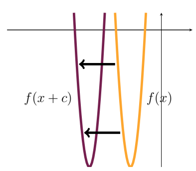
Translations

Horizontal Translations:

- The graph of $f(x-c)$ is $f(x)$ shifted c units to the right.

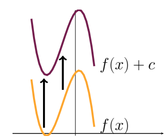


- The graph of $f(x+c)$ is $f(x)$ shifted c units to the left.

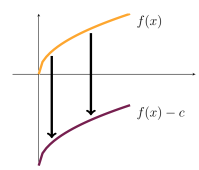


Vertical Translations:

- The graph of $f(x)+c$ is $f(x)$ shifted c units up.

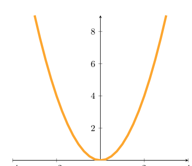


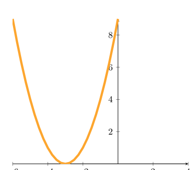
- The graph of $f(x)-c$ is $f(x)$ shifted c units down.



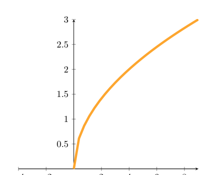
Examples

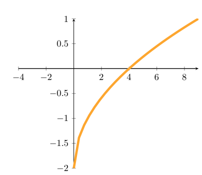
1. Graph $y = x^2$ and $y = (x+3)^2$.





2. Graph $y = \sqrt{x}$ and $y = \sqrt{x} - 2$

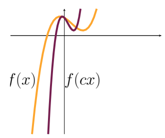


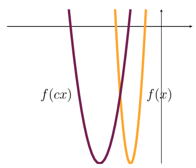


Stretching/Shrinking

Horizontal Stretch/Shrink:

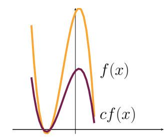
- If c is positive, $f(cx)$ is $f(x)$ stretched or shrunk by a factor of $\frac{1}{c}$ from/towards the y-axis.

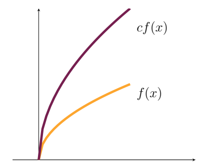




Vertical Stretch/Shrink:

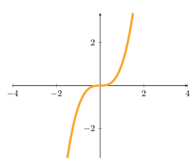
- If c is positive, $cf(x)$ is $f(x)$ stretched or shrunk by a factor of c from/towards the x-axis.

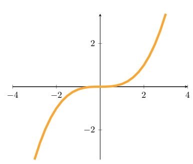




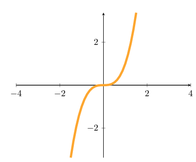
Examples

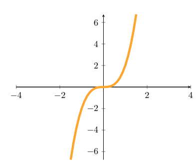
1. Graph $y = x^3$ and $y = (\frac{1}{2}x)^3$.





2. Graph $y = x^3$ and $y = 2x^3$.

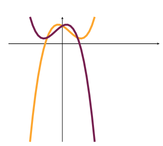




Reflection

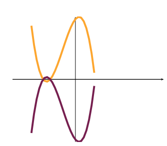
Horizontal Reflection:

- $f(-x)$ is $f(x)$ reflected across the y-axis.

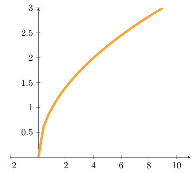


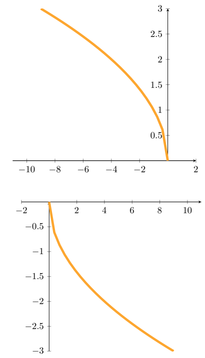
Vertical Reflection:

- $-f(x)$ is $f(x)$ reflected across the x-axis.

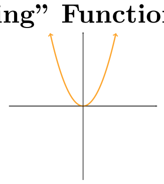


Examples
Graph $y = \sqrt{x}$ and $y = \sqrt{-x}$ and $y = -\sqrt{x}$.

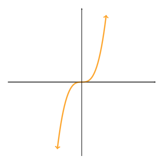




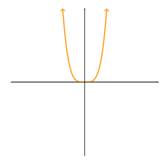
"Starting" Functions



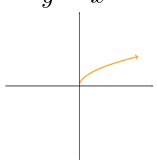
$y = x^2$



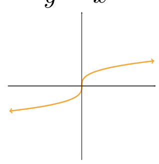
$y = x^3$



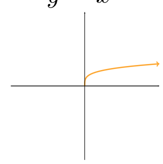
$y = x^4$



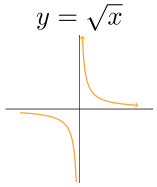
$y = \sqrt{x}$



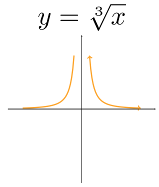
$y = \sqrt[3]{x}$



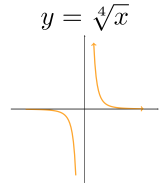
$y = \sqrt[4]{x}$



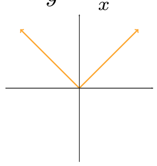
$y = \frac{1}{x}$



$y = \frac{1}{x^2}$



$y = \frac{1}{x^3}$



$y = |x|$

Graphing Equations with Multiple Transformations

1. Identify the "starting" function.
$$y = \sqrt{x}$$

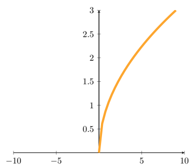
2. Compare the "starting" function with the final function. Anything that's changed where the x is in the starting function will correspond to a horizontal transformation. Anything else will correspond to a vertical transformation.

Vertical

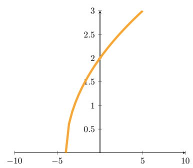
$$y = \sqrt{\boxed{x}} \qquad \text{vs} \qquad y = \boxed{\frac{1}{2}}\sqrt{\boxed{4 - x}} + \boxed{1}$$

Horizontal

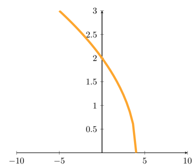
3. Graph the horizontal transformations, by going *against* the order of operations.



$y = \sqrt{x}$



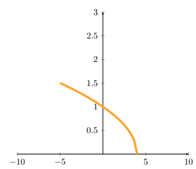
$y = \sqrt{x+4} = \sqrt{4+x}$



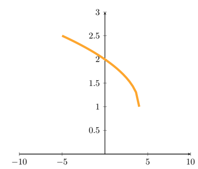
$y = \sqrt{4-x}$

We'll use $y = \frac{1}{2}\sqrt{4-x} + 1$ to illustrate.

4. Graph the vertical transformations, by following the order of operations.



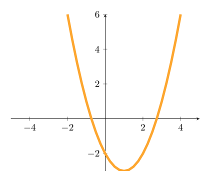
$y = \frac{1}{2}\sqrt{4-x}$



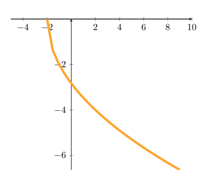
$y = \frac{1}{2}\sqrt{4-x} + 1$

Examples

1. Graph $y = (x-1)^2 - 3$



2. Graph $y = -2\sqrt{x+2}$



2 Symmetry of Functions

Even and Odd Functions

- A function f is *even* if $f(-x) = f(x)$.

  These functions are symmetric across the y-axis.

- A function f is *odd* if $f(-x) = -f(x)$.

  These functions are symmetric about the origin.

Check if the function is even, odd, or neither.

1. $f(x) = x^3 - \frac{1}{x}$

$\boxed{\text{odd}}$

2. $f(x) = x^4 + 4x^2 - 3$

   $\boxed{\text{even}}$

3. $f(x) = x^3 - 2x^2$

   $\boxed{\text{neither}}$