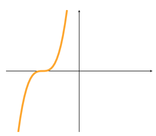
Section 2.7 Notes

One-to-One Functions and Their Inverses

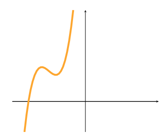
1 One-To-One Functions

Definition

- A *one-to-one* (or *1-1*) function has the property that no two x-values (inputs) have the same y-value (output).
- Graphically, one-to-one functions pass the *Horizontal Line Test*: a function passes the horizontal line test if every horizontal line intersects the graph at most once.



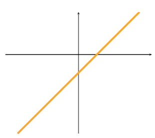
One-to-One



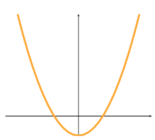
Not One-to-One

Examples

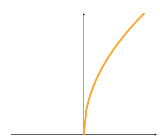
Which graphs correspond to one-to-one functions?

1. 

   One-to-One

2. 

   Not One-to-One

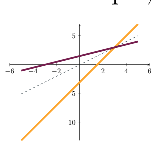
3. 

   One-to-One

2 Inverse Functions

Definition

- The *inverse function* of a function f is the function that switches all the inputs and outputs. In other words, if (a, b) is a point on the graph of f, then (b, a) is a point of the graph of f^{-1}.
- Graphically, f and f^{-1} are reflections of each other across the diagonal line $y = x$.
- For example, consider $f(x) = 2x - 3$:

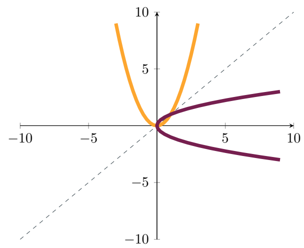


x	$f(x)$
-2	-7
0	-3
2	1

x	$f^{-1}(x)$
-7	-2
-3	0
1	2

Functions without Inverses

- Not every function has an inverse function, however: Consider $f(x) = x^2$



Notice how the "inverse function" is not actually a function - it fails the vertical line test!

- Only *one-to-one* functions have inverses.

Properties of Inverse Functions

- Inverse Function Property (used to show that two functions are inverses): If f and g are inverses of each other, then
  - $(f \circ g)(x) = x$, and
  - $(g \circ f)(x) = x$.
- If f and g are inverses, then their domain and range are switched:
  - Domain of f = Range of g
  - Range of f = Domain of g

Example
Show that f and g are inverses of each other: $f(x) = 2x + 1$ and $g(x) = \frac{1}{2}x - \frac{1}{2}$.

Finding the Formula for an Inverse Function

1. Switch x and y (if the function is written as $f(x)$, treat that like "y").
2. Solve for y.
3. If necessary, go back to function notation - "y" becomes $f^{-1}(x)$.

Examples
Find the inverse of each function.

- $f(x) = x^3 + 3$

  $\boxed{f^{-1}(x) = \sqrt[3]{x-3}}$

- $f(x) = \frac{2x-1}{x+3}$

  $\boxed{f^{-1}(x) = \frac{1+3x}{2-x}}$

- $f(x) = \sqrt{x} + 1$

  $\boxed{f^{-1}(x) = (x-1)^2, \text{ where } x \geq 1}$